SUPPLEMENT

TO BRITISH TELECOMMUNICATIONS ENGINEERING

(formerly the Supplement to The Post Office Electrical Engineers' Journal)

Vol. 2 Part 1 April 1983

ISSN 0262-4028

TEC & SCOTEC
GUIDANCE FOR STUDENTS

Contents

TECHNICIAN EDUCATION COUNCIL

Certificate Programme in Telecommunications

Sets of model questions and answers for Technician Education Council (TEC) units are given below. The questions illustrate the types of questions that students may encounter, and are useful as practice material for the skills learned during the course.

Where additional text is given for educational purposes, it is shown within square brackets to distinguish it from information expected of students under examination conditions. Where possible, representative time limits or marks are shown for each question, and care has been taken to give model answers that reflect these limits.

We would like to emphasise that the questions are not representative of questions set by any particular college.

TEC: LINE AND CUSTOMER APPARATUS I

TEC unit number U81/749. Students are advised to read the notes above

A method of assessment that is sometimes used is demonstrated by this paper. Students might be asked to complete all of the questions in Section A and 3 out of the 4 questions in Section B. Students are advised to allow approximately 60 min for each section. Each question in Section A has a maximum of 5 marks, and each question in Section B has a maximum of 20 marks.

SECTION A

Q1 List 5 examples of the domestic and social uses of a telephone.

A1 Any 5 of the following typical examples:

(*a*) The telephone allows the family to keep in touch with friends.

(*b*) The telephone allows shopping to be done from home.

(*c*) The telephone can be used to contact the emergency services quickly.

(*d*) The telephone allows members of the family and friends to be contacted during working hours.

(*e*) The telephone can be a life-line for some people; for example senior citizens and invalids.

(*f*) The telephone can be used to sell unwanted goods from home by using an advertised telephone number.

Q2 Briefly explain the need for a telephone exchange.

A2 A telephone exchange provides the means by which telephone lines can be interconnected. Connections can be made between 2 lines, or a line and a function, by means of electromechanical or electronic equipment within the exchange. While the connection is made, information can be passed between the 2 customers. When the call is finished the exchange equipment releases the connection.

Q3 What is the cost of a telephone call between 2 customers, 75 km apart, lasting 4 min, if the charge rates in the following table apply?

Type of call	*Charge rate: standard. Time allowed for unit charge of 4·3p, excluding value added tax (VAT)*
Local calls	*3 min*
Calls up to 56 km distance	*45 s*
Calls over 56 km distance	*15 s*

VAT applies at 15%.

A3 A call over a distance of 75 km falls within the "calls over 56 km distance" category, with 15 s allowed for a unit charge.

For a call of 4 min, the number of units charged,

$$= \frac{4 \times 60}{15} = 16.$$

The total cost of 16 units, excluding VAT,

$$= 16 \times 4{\cdot}3 = 68{\cdot}8\text{p}.$$

$$\text{VAT at } 15\% = 68{\cdot}8 \times \frac{15}{100} = 10{\cdot}32\text{p}.$$

$$\text{Total cost} = 68{\cdot}8 + 10{\cdot}32 = \underline{79{\cdot}12\text{p} \approx 79\text{p}.}$$

Q4 (a) List 4 means of attracting the attention of a called customer.

(b) For each of the methods given in the answer to part (a), name a different situation where it is used.

A4 (*a*) Four means of attracting the attention of a called customer are

(*i*) a bell calling signal,
(*ii*) a buzzer calling signal,
(*iii*) a tone calling signal, and
(*iv*) a lamp calling signal.

(*b*) The situations in which each of the above methods are used are, for example,

(*i*) telephones (bells),
(*ii*) switchboards (buzzer),
(*iii*) Trimphone (tone caller), and
(*iv*) lamp signalling handset.

[*Tutorial Note:* For answer (*b*) the following are suitable alternatives, as appropriate: loudspeaking telephones, extension telephones, extension bells, key-and-lamp units, and lamps in the body of a telephone.]

Q5 Which of the following actions can be applied to a line to give an answering signal (tick the correct answers):

(*a*) *replacing the handset on a telephone,*
(*b*) *operating a key on a PMBX,*
(*c*) *removing the handset from a telephone,*
(*d*) *dialling the digit 0,*
(*e*) *restoring a key on a PMBX to normal, or*
(*f*) *pressing a key on a keypad?*

A5 (*b*) and (*c*)

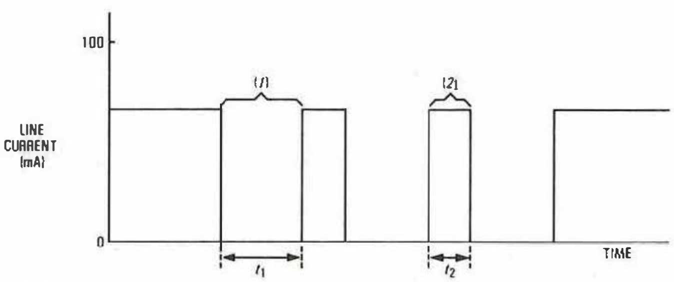
Q6 Explain what is meant by the term "grade of service" as applied to exchange switching equipment.

A6 Exchange switching equipment is provided in sufficient quantity to carry most, but not all, of the telephone traffic during the busy hour. The quality of service provided is known as the grade of service. The grade of service is expressed in terms of the calls offered and those allowed to fail because of insufficient exchange switching equipment, during the busy hour, as follows:

$$\text{grade of service} = \frac{\text{number of calls that fail}}{\text{number of calls offered}}.$$

Q7 The sketch represents the coded electrical signals applied to a line by a telephone dial.

(*a*) *Name the type of pulse represented by (1) and (2).*
(*b*) *State the time period of t_1 and t_2, assuming that the dial speed and ratio are correct.*
(*c*) *What digit has been dialled?*



A7 (*a*) (*1*) *Break* pulse
(*2*) *Make* pulse

(*b*) If the dial has a correct speed of 10 pulses/s and a ratio of 2:1, then $t_1 = 66{\cdot}67$ ms and $t_2 = 33{\cdot}33$ ms.

(*c*) Since there are 3 *break* pulses, the digit 3 has been dialled.

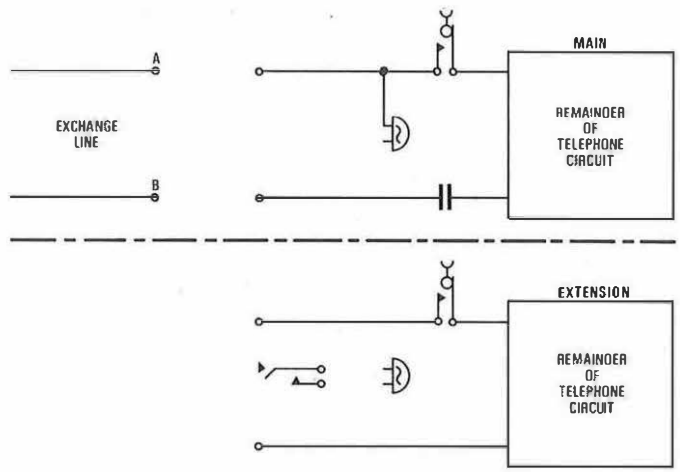
Q8 Explain what is meant by the following facilities provided by a PABX:

(*a*) *first party clear; and*
(*b*) *enquiry and transfer.*

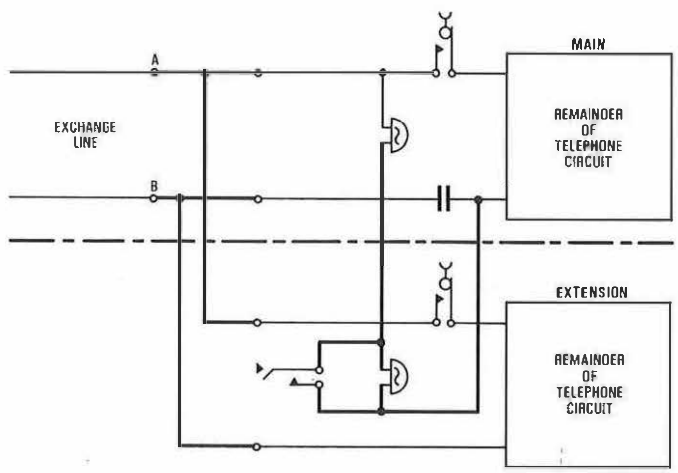
A8 (*a*) The connection between 2 extensions on a PABX with the first-party-clear facility is cleared by either the calling or called extension clearing the line. This avoids the unnecessary holding of the PABX equipment and the called party.

(*b*) With the enquiry-and-transfer facility an extension can hold an exchange call while making an enquiry to another extension and, if required, transfer the exchange line to the other extension.

Q9 The sketch shows a simplified telephone extension arrangement with several connections missing. Complete the connections.



A9



[*Tutorial Note:* The sketch in the answer shows

(*a*) telephones connected in parallel,
(*b*) bells connected in series, and
(*c*) the capacitor in the bell circuit.]

Q10 List the primary coefficients of a cable and state the SI units in which they are expressed.

A10 The primary coefficients are

(*a*) resistance, R, in ohms (Ω),
(*b*) inductance, L, in henrys (H),
(*c*) capacitance, C, in farads (F), and
(*d*) conductance, G, in siemens (S).

Q11 *"Cast iron is used extensively to protect high-voltage switching equipment and underground repeaters."*
Explain what special precautions for electrical safety are taken, and why, when cast iron is used in this way.

A11 Any iron that is in close proximity to electricity is bonded together and earthed so that in the event of a fault or accident the risk of electrical shock to personnel or public is reduced. If the fault causes contact between the electricity supply and the metal case, then the bonding ensures that the current flows to earth through the case and not through the persons body. This current should trip a cut-out or blow a fuse to disconnect the supply.

Q12 (*a*) *State the difference between using direct labour and contract labour to carry out project work.*

(*b*) *Give 2 reasons why project work should be carried out by*

(*i*) *direct labour, and*
(*ii*) *contract labour.*

A12 (*a*) Direct labour is carried out by the business's own employees, whereas contract labour is hired from another firm.

(*b*) (*i*) Direct labour is used where work has to be done regularly. Also, personnel will be familiar with plant and work procedures.

[*Tutorial Note:* Other reasons are as follows:
Direct-labour work can be programmed and controlled directly by the business.
Direct labour can be used to carry out more than one type of job on the project.]

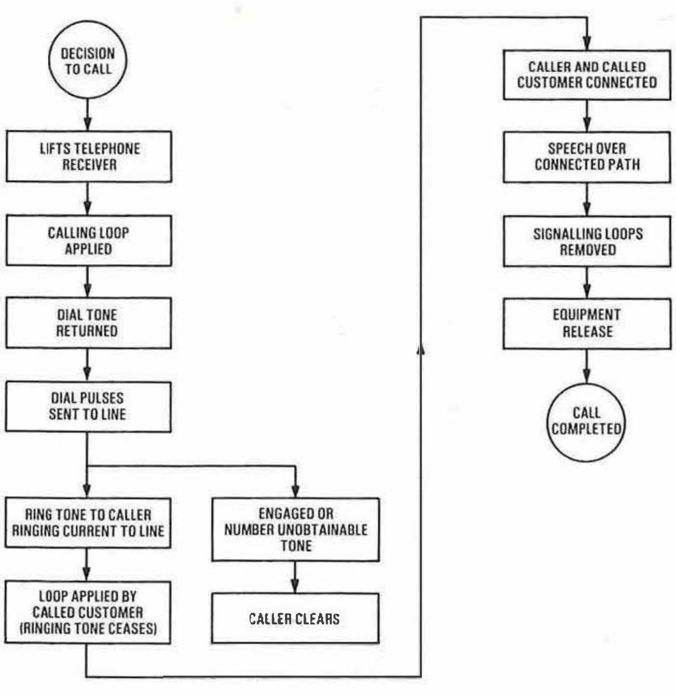
(*ii*) Contract labour is used for work that is not done regularly. Also, contract labour is used where the business is not equipped to do a certain type of work.

[*Tutorial Note:* Other reasons are as follows:
Contract labour is used when the business has too much work to handle on its own.
Contractors may already be carrying out similar work on the same development.]

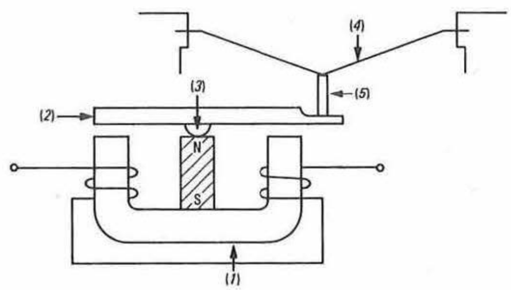
SECTION B

Q13 *By means of a block diagram (flow chart) show the signalling operations for an automatically connected call via an automatic exchange. Carefully label the blocks to indicate all the caller's actions and all the signals to indicate the progress of the call.*

A13



Q14 (*a*) *The sketch shows a rocking armature receiver. Identify each of the parts labelled (1) to (5).*



(*b*) *Indicate on the diagram the magnetic flux path due to*

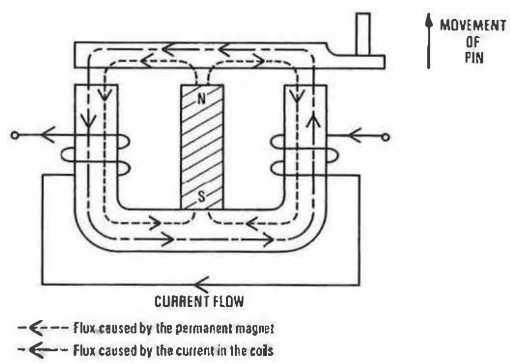
(*i*) *the permanent magnet, and*
(*ii*) *the current in the speech coils.*

In both cases indicate the direction of the flux.

(*c*) *Explain how the rocking armature receiver converts electrical signals into sound waves.*

A14 (*a*) (*1*) Soft iron core.
(*2*) Rocking armature.
(*3*) Pivot.
(*4*) Diaphragm.
(*5*) Driving pin.

(*b*)



[*Tutorial Note:* An alternative answer would be to show the current flow in the opposite direction. In this case the flux caused by the current flow should be shown in the opposite direction and the pin shown as moving downwards.]

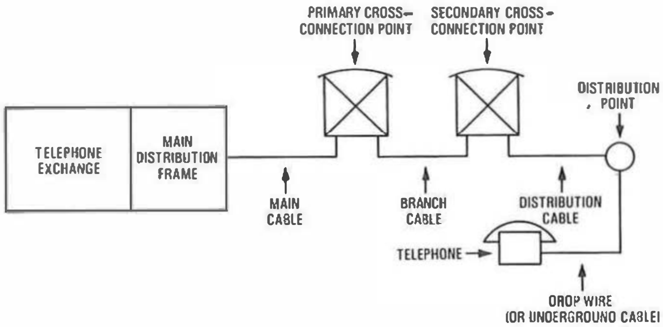
(*c*) The permanent magnet provides a polarising flux as indicated in the sketch in the answer to part (*b*). Current flowing in the speech coils produces a second magnetic flux, in the path shown, which reacts with the polarising flux in such a way that the force of attraction on the armature is increased at one side and decreased at the other. In the sketch the direction of current is such that the flux is increased in the air gap on the left, and weakened on the right. Therefore, the armature rocks down on the left and up on the right. The driving pin moves upwards and the diaphragm is pushed forwards (upwards in the sketch), compressing the air. A reversal of current in the coils produces the opposite effects to produce a rarefication of the air in front of the diaphragm. If an alternating speech current is passed through the coil, the receiver generates sound waves in sympathy with the speech current.

Q15 (*a*) *Draw a labelled diagram of the local distribution network from the exchange to the customer.*

(*b*) *List the flexibility points in the local distribution network.*

(*c*) *List 2 metals that are used in the manufacture of the cables used for local distribution. Briefly explain why each metal listed is suitable for its particular application.*

A15 (*a*)



(*b*) The flexibility points in the local distribution network are:

(*i*) the main distribution frame (MDF) in the exchange,
(*ii*) the primary cross-connection point,
(*iii*) the secondary cross-connection point,
(*iv*) the distribution point.

(*c*) Two metals that are used in the manufacture of cables in the local distribution network are

(*i*) copper as a conductor, and
(*ii*) aluminium as a conductor or sheath material.

These metals are used for the following reasons:

(*i*) *Copper* Copper is very malleable and can be cold drawn down into wire; the wire can be bent and twisted without it easily fracturing. Copper can be joined easily by soldering and it has a good resistance to corrosion. Of course, copper is a very good conductor of electricity (that is, it has a high conductivity).

(*ii*) *Aluminium* In recent years the supply and price of copper has been unstable, so that aluminium has been used as an alternative conductor for cables. Aluminium is a good conductor and is widely available. Although aluminium is not as heavy as copper, it has a lower conductivity, which means that a larger cable has to be used to obtain similar electrical properties.

The handling characteristics of the first aluminium cables were not very good because of the materials poor ductility and elongation. However, alloying aluminium with iron, silicon and other elements has greatly improved the handling properties. Aluminium conductors are difficult to joint by soldering and, consequently, a method of crimping is normally used.

Aluminium in a corrugated form is sometimes used for cable sheaths in special cases where a high degree of screening is required.

[*Tutorial Note:* Lead was widely used as a sheath material because it has a reasonable cost, is flexible, can be jointed easily, and is able to withstand pulling tensions. However, with the introduction of polyethylene-sheathed cables, which do not corrode, the demand for lead-sheathed cables has diminished, and they are no longer used in the local distribution network.]

Q16 *(a) What is the purpose of carrying out a preliminary survey?*
(b) What particular information is noted during the survey?
(c) For what purpose is the information used?
(d) State the reasons for programming work.

A16 (*a*) The purpose of the preliminary survey is to check the information used in the desk proposals.

(*b*) The following information is carefully noted during the preliminary survey:

(*i*) any obstructions on the proposed route,
(*ii*) the proximity of any high-voltage lines with respect to the proposed route,
(*iii*) information from local authorities and other bodies controlling highways and land,
(*iv*) the number of poles required for any overhead sections of the route, and
(*v*) the space available in ducts and joint boxes on the underground sections.

(*c*) The information obtained during the preliminary survey is used to determine the most suitable route and then to draw up a basic plan of the project.

(*d*) The reasons for programming work are:

(*i*) *Efficiency* To ensure that the work is completed as efficiently as possible.
(*ii*) *Organisation* To ensure that the project resources can be properly organised.
(*iii*) *Progress* To enable the progress of the project to be monitored.
(*iv*) *Control* To enable the project resources to be controlled.

R. Wilson

TEC: DIGITAL TECHNIQUES II

TEC unit number U81/750: Students are advised to read the notes on p. 1

Q1 *Convert each of the following denary numbers into its binary equivalent and give the 1's complement and 2's complement.*

(a) 131,
(b) 78,
(c) 13¾,
(d) 5·625, and
(e) 17·375.

A1 The final answers are given in the following table.

	Denary Number	Binary Equivalent	1's Complement	2's Complement
(*a*)	131	10 000011	01 111 100	01 111 101
(*b*)	78	1 001 110	0 110 001	0 110 010
(*c*)	13¾	1101·11	0010·00	0010·01
(*d*)	5·625	101·101	010·010	010·011
(*e*)	17·375	10 001·011	01 110·100	01 110·101

[*Tutorial Note:* The working for part (*e*) is given below to illustrate the general method for all parts of this question.

The binary representation of the integral part of 17·375 is obtained by repeatedly dividing the integral part by 2 and noting the remainder, as shown in the following table.

Integral Part	
Quotient	Remainder
2)17	
8	1
4	0
2	0
1	0
0	1

The binary representation of the fractional part of 17·375 is obtained by repeatedly multiplying the fractional by 2 and noting the integral part of the result, as shown in the following table.

Fractional Part	
Result	Product
	0·375 × 2
0	0·75
1	0·5
1	0·0

For the integral part, the remainder is noted in reverse order (the final remainder being the most significant digit). For the fractional part, the result is noted in correct order.

$\therefore \quad 17{\cdot}375_{10} = 10\,001{\cdot}011_2.$

The answer can be verified by adding the appropriate powers of 2 as follows:

1	0	0	0	1 ·	0	1	1
×	×	×	×	×	×	×	×
2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
16	+0	+0	+0	+1	+0	+0·25	+0·125 = 17·375.

As some binary fractions never work out finally, a limit can be placed on the number of binary digits to the right of the decimal point (as in denary numbers). However, the syllabus for TEC U81/780 limits fractions of denary numbers for conversion to $\frac{1}{32}$. These numbers can be represented exactly in binary form with up to 5 binary digits to the right of the decimal point.

The 1's complement of a binary number is obtained by inverting each binary digit of the original number. The 2's complement is obtained by finding the 1's complement and adding 1.

Hence,

17·375:	10 001·011
1's complement:	01 110·100
Add 1:	1+
2's complement:	01 110·101

Note that for a fractional binary number, the 2's complement is found by adding 1 to the 1's complement at the least significant fractional digit.]

Q2 Add together 101·11₂ and 10 100·1011₂ and convert the result into a denary number with fractions.

A2

00 101·1100
10 100·1011+
11 010·0111

The result is converted into denary as follows:

1	1	0	1	0 ·	0	1	1	1
×	×	×	×	×	×	×	×	×
2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
16	+8	+0	+2	+0	+0	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{1}{16} = 26\frac{7}{16}$.

Q3 In digital computers, binary subtraction is performed by using the 2's-complement method. Demonstrate this by carrying out the following calculations in binary arithmetic, where $A = 111\,000_2$, $B = 1\,101_2$ and $C = 10\,001_2$. Convert the binary results into denary.

(*a*) *A–B*, *and*

(*b*) *C–A*.

A3 (*a*) [*Tutorial Note:* The given binary numbers can be stored as 7-bit binary numbers, where the most significant bit is used as a sign bit. The sign bit is 0 for a positive number and 1 for a negative number. The calculation $A - B$ can be performed by creating the negative of B and adding to A; that is $A - B = A + (-B)$.]

A negative binary number is formed by taking the 2's complement, as follows.

	Sign bit	
B:	0	001 101
1's complement:	1	110 010
Add 1:		1+
2's complement:	1	110 011

The solution is found by adding the 2's complement of *B* to *A*, as follows.

		Sign bit	
A:		0	111 000
2's complement of *B*:		1	110 011+
	1	0	101 011

The most significant bit is lost, since this is outside the maximum number of bits used. Therefore, the binary result is 101 011. (The sign bit is 0, therefore the result is positive.)

The binary result is converted into denary as shown below.

1	0	1	0	1	1
×	×	×	×	×	×
2^5	2^4	2^3	2^2	2^1	2^0
32	+0	+8	+0	+2	+1 = 43.

(*b*) Similarly, the calculation $C - A$ is shown below.

	Sign bit	
A:	0	111 000
1's complement of *A*:	1	000 111
Add 1:		1+
2's complement of *A*:	1	001 000
Add *C*:	0	010 001
	1	011 001

The sign bit indicates that the result is negative. Conversion to a negative number is performed by subtracting 1 to obtain the 1's complement, and inverting each bit, as follows.

2's complement	011 001
1's complement	011 000
Inversion	100 111

The binary result is therefore −100 111. This is converted into denary as shown below.

1	0	0	1	1	1
×	×	×	×	×	×
2^5	2^4	2^3	2^2	2^1	2^0
32	+0	+0	+4	+2	+1 = 39.

Therefore, the denary result is −39.

Q4 Multiply 54_{10} by 25_{10} by using binary arithmetic. Your working should demonstrate how this can be performed by successively shifting and adding. Convert your answer into denary form.

A4 The binary equivalent of 54_{10} and 25_{10} is found by successively dividing by 2 and noting the remainders, as follows.

54_{10}	
Quotient	Remainder
2)54	
26	0
13	1
6	1
3	0
1	1
0	1

25_{10}	
Quotient	Remainder
2)25	
12	1
6	0
3	0
1	1
0	1

The binary numbers are obtained by writing the remainders down in reverse order.

Hence, $54_{10} = 110\,110_2$, and
$25_{10} = 11\,001_2$.

The multiplication is performed, by successively writing down the multiplicand shifted to the left in accordance to the power of the multiplier digit, as follows.

Multiplicand (54_{10}):	110 110
Multiplier (25_{10}):	011 001 ×
Copy multiplicand, shifted 0 places:	110 110+
Copy multiplicand, shifted 3 places:	110 110 000+
Copy multiplicand, shifted 4 places:	1 101 100 000+
Product ($54_{10} \times 25_{10}$):	10 101 000 110

The binary result is 10 101 000 110_2.
Converting to denary:

1	0	1	0	1	0	0	0	1	1	0
×	×	×	×	×	×	×	×	×	×	×
2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

1024 +0 +256 +0 +64 +0 +0 +0 +4 +2 +0 = 1350.

Q5 Divide 10 100 101_2 by 1111_2, by using binary division.

A5 The numbers can be divided by using a shift and subtract technique, as follows.

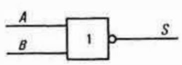
```
              1 0 1 1
          ____________________
1 1 1 1)1 0 1 0 0 1 0 1
        1 1 1 1
        ____________
        0 0 1 0 1 1 0
            1 1 1 1
            ________
            0 0 1 1 1 1
                1 1 1 1
                _______
                0 0 0 0
```

Thus, the binary result is 1011.

Q6 (a) Draw the British Standards (BS) symbol for a NOR gate, and give the truth table for the gate, if the inputs are A and B and the output is S.

(b) Show how a NOR function can be realised in an electrical circuit by using simple relay contacts.

A6 (*a*) The BS symbol for a NOR gate is shown in sketch (*a*).

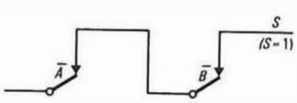


(*a*)

The truth table is as follows.

A	*B*	*S*
0	0	1
0	1	0
1	0	0
1	1	0

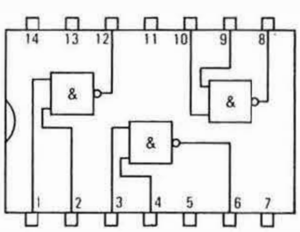
(*b*) A NOR function can be realised by 2 *break* contacts connected as shown in sketch (*b*). The circuit is broken by either one or both of the contacts operating.



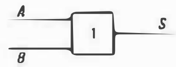
(*b*)

Q7 (a) Draw the British Standards (BS) symbol for an OR gate. Give the truth table for the gate and write down the Boolean expression if the inputs are A and B and the output is S.

(b) Connect the numbered pins of the integrated circuit shown in the sketch to construct an OR gate. The output S is to be at pin 8.



A7 (*a*) The BS symbol for an OR gate is shown in sketch (*a*).



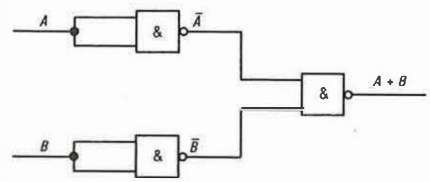
(*a*)

The truth table is shown below.

A	*B*	*S*
0	0	0
0	1	1
1	0	1
1	1	1

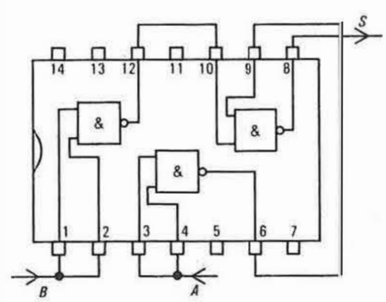
The Boolean expression for an OR gate is $S = A + B$.

(*b*) The integrated circuit consists of 3 NAND gates. An OR gate can be constructed from NAND gates connected as shown in sketch (*b*).



(*b*)

This circuit can be constructed by joining the pins on the integrated circuit as shown in sketch (*c*).



(*c*)

[*Tutorial Note:* The following alternative connections are also possible:

(*a*) connecting *A* to pins 1 and 2, and *B* to pins 3 and 4; and

(*b*) connecting pin 6 to pin 10 and pin 12 to pin 9.]

Q8 A payphone coinbox accepts 5p and 10p coins. A maximum of three 5p coins and two 10p coins can be inserted to make a call. Signals are generated to indicate which coins have been inserted, and these signals may be represented by the Boolean variables shown in the following table.

When a coin is inserted the appropriate Boolean variable is set to 1.

Coins	*Boolean variable*
First 5p coin inserted	A_1
Second 5p coin inserted	A_2
Third 5p coin inserted	A_3
First 10p coin inserted	B_1
Second 10p coin inserted	B_2

(a) Construct a truth table showing the states of the Boolean functions for each of the combinations of coins that can be inserted for total values of between 10p and 25p, inclusively.

(b) Four signals, represented by the Boolean functions S_{10p}, S_{15p}, S_{20p} and S_{25p}, indicate to a control circuit the total value of coins inserted (for example, S_{10p} indicates coins with a total of 10p). Write down Boolean expressions for S_{10p}, S_{15p}, S_{20p} and S_{25p} in terms of A_1, A_2, A_3, B_1 and B_2. It does not matter if for particular total values, signals for lower total values are also set.

(c) Draw an arrangement of logic gates that generates the required signal when 20p is inserted.

A8 (*a*) The truth table is shown below.

Coins inserted		Total value	Variables				
5p	10p		A_1	A_2	A_3	B_1	B_2
0	1	10p	0	0	0	1	0
2	0		1	1	0	0	0
1	1	15p	1	0	0	1	0
3	0		1	1	1	0	0
0	2	20p	0	0	0	1	1
2	1		1	1	0	1	0
1	2	25p	1	0	0	1	1
3	1		1	1	1	1	0

(*b*)

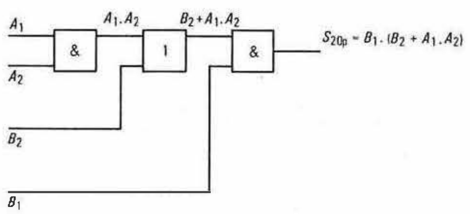
$$S_{10p} = \underline{B_1 + A_1.A_2.}$$

$$S_{15p} = A_1.A_2.A_3 + A_1.B_1,$$
$$= \underline{A_1.(A_2.A_3 + B_1).}$$

$$S_{20p} = B_1.B_2 + A_1.A_2.B_1,$$
$$= \underline{B_1.(B_2 + A_1.A_2).}$$

$$S_{25p} = A_1.B_1.B_2 + A_1.A_2.A_3.B_1,$$
$$= \underline{A_1.B_1.(B_2 + A_2.A_3).}$$

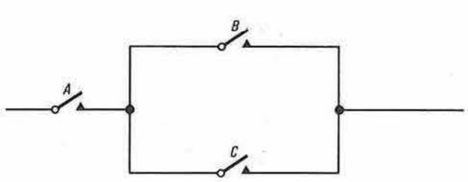
(*c*) The logic diagram for S_{20p} is shown in the sketch.



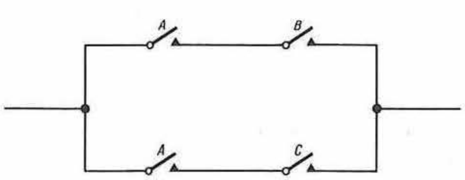
Q9 State an equivalent Boolean function to A.(B + C) and draw diagrams showing how both of these functions can be represented by relay contacts.

A9 $A.(B + C) = A.B + A.C$

The circuit representing $A.(B + C)$ is shown in sketch (*a*), and that representing $A.B + A.C$ is shown in sketch (*b*).



(*a*)



(*b*)

Q10 Consider the following 5 truth tables ((a)–(e)) for logic gates. In the table provided draw the British Standards symbol for the gates and state their names and functions in terms of Boolean algebra.

(*a*)

J	K	M
0	0	1
0	1	0
1	0	0
1	1	0

(*b*)

N	P	Q
0	0	1
0	1	1
1	0	1
1	1	0

(*c*)

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

(*d*)

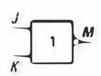
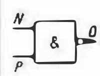
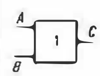
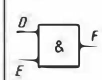
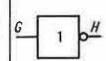
D	E	F
0	0	0
0	1	0
1	0	0
1	1	1

(*e*)

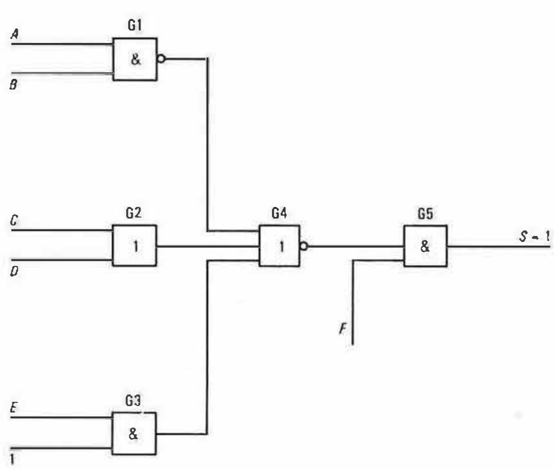
G	H
0	1
1	0

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
BS Symbol					
Name					
Boolean function	*M* =	*Q* =	*C* =	*F* =	*H* =

A10

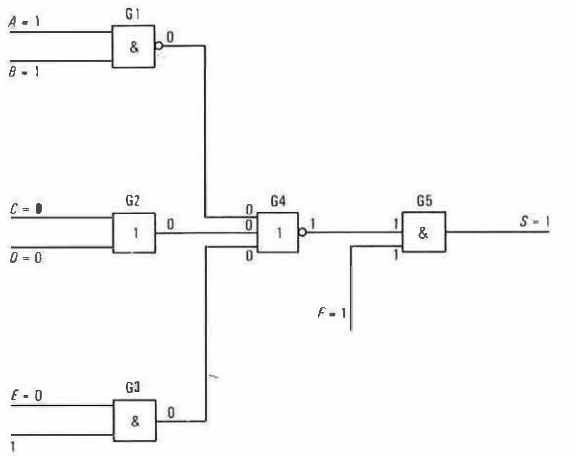
	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
BS Symbol					
Name	NOR gate	NAND gate	OR gate	AND gate	NOT gate
Boolean function	$M = \overline{J + K}$	$Q = \overline{N.P}$	$C = A + B$	$F = D.E$	$H = \overline{G}$

Q11 For the logic diagram shown in the sketch, what are the values of A, B, C, D, E, and F if S = 1. Mark these values and the states of each output on the diagram.



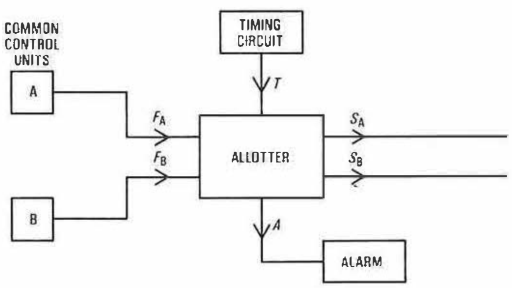
A11 See the sketch.

Working back from the output, as $S = 1$ and gate G5 is an AND gate,



both the inputs to gate G5 must be 1. Considering gate G4, the output of a NOR gate can only be 1 if all the inputs are 0; therefore, all the inputs to gate G4 are 0. As gate G1 is a NAND gate, both A and B must be 1, and, as gate G2 is an OR gate, both C and D must be 0. As one input of gate G3 is set to 1, E must be set to 0 for the output of gate G3 to be 0.

Q12 *Refer to the sketch. An exchange is provided with 2 common control units (CCUs), A and B, for security reasons. The allotter controls which CCU is to be used for a call, by sending signals on outputs S_A and S_B, depending on the states of the inputs, F_A, F_B and T.*



The functions of the inputs and outputs to the allotter are described below.

(a) Input T — If the 2 CCUs are working correctly, then they are to be used alternately. This is controlled by a timing signal, T, which is set to state 0 to indicate that CCU A is to be used or state 1 to indicate that CCU B is to be used.

(b) Inputs F_A and F_B — These inputs indicate whether the CCUs are faulty: if a CCU becomes faulty, then the appropriate input, F_A or F_B, is set to state 1. If one CCU is faulty then the allotter must ensure that the other CCU is used, regardless of the state of the timing signal T.

(c) Outputs S_A and S_B — To indicate which CCU is to be used, the allotter must set S_A or S_B to state 1, as appropriate.

(d) Output A — Output A controls an alarm. Should both CCUs become faulty, then A must be set to logic 1 to control the alarm.

From the above information,

(a) Complete the truth table for the operation of the allotter, shown below.

Inputs			*Outputs*		
F_A	F_B	T	S_A	S_B	A
0	0	0			
0	1	0			
1	0	0			
1	1	0			
0	0	1			
1	0	1			
0	1	1			
1	1	1			

(b) Write down the Boolean expressions for S_A, S_B and A.

(c) Draw a logic diagram showing how S_A, S_B and A can be obtained by using logic gates.

A12 (*a*)

Inputs			Outputs		
F_A	F_B	T	S_A	S_B	A
0	0	0	1	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1
0	0	1	0	1	0
1	0	1	0	1	0
0	1	1	1	0	0
1	1	1	0	0	1

(*b*) The Boolean expressions are derived as follows:

$$S_A = \overline{T}.\overline{F_A} + T.F_B.\overline{F_A},$$
$$= \overline{F_A}.(\overline{T} + T.F_B),$$
$$= \overline{F_A}.(\overline{T} + F_B).$$

[*Tutorial Note:* The last step can be shown as follows:

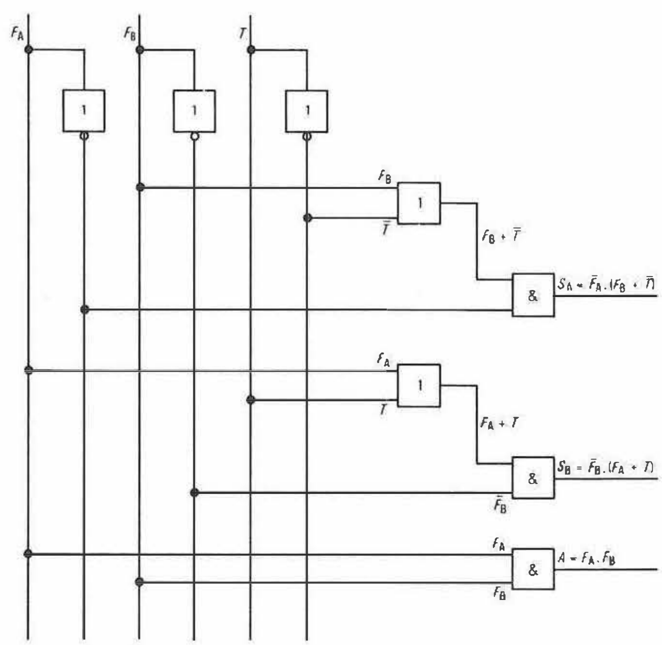
As $\quad A + B.C = (A + B).(A + C),$

then, $\quad \overline{T} + T.F_B = (\overline{T} + T).(\overline{T} + F_B),$

$= \overline{T} + F_B.$]

$$S_B = T.F_B + F_A.\overline{F_B}.\overline{T},$$
$$= \overline{F_B}.(T + \overline{T}.F_A),$$
$$= \overline{F_B}.(T + F_A).$$
$$A = F_A.F_B.$$

(*c*) The logic diagram is shown in the sketch.



A. J. Hinkly

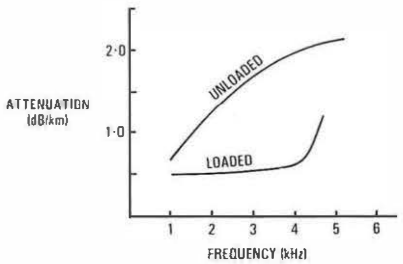
Q1 Sketch and label the following:

(a) typical attenuation/frequency curves, and

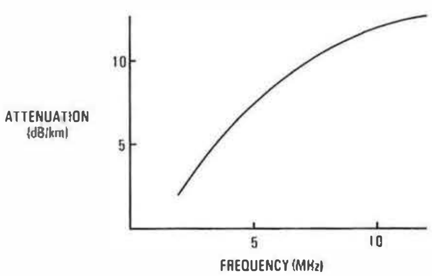
(b) typical group-delay/frequency curves for the following types of cables:

(i) unloaded audio,
(ii) loaded audio, and
(iii) coaxial. (*10 min*)

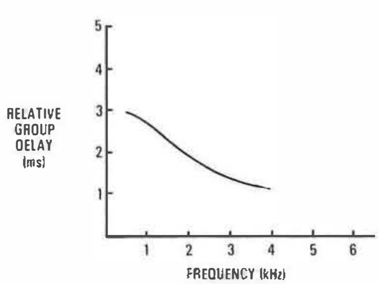
A1 (*a*) (*i*) (*ii*)



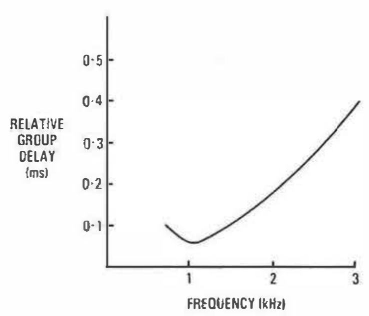
(*iii*)



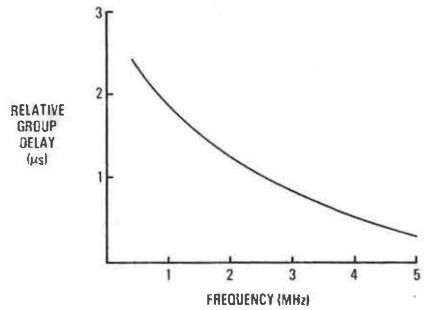
(*b*) (*i*)



(*ii*)



(*iii*)



Q2 Indicate whether the following statements regarding the characteristics of cables are true or false.

(a) A graph of power loss against frequency is described as a group-delay/frequency curve. *TRUE/FALSE*

(b) The attenuation/frequency responses of both loaded and unloaded audio and of coaxial pairs show a gradual increase in the rate of rise of attenuation with frequency. *TRUE/FALSE*

(c) All losses in cables increase with distance. *TRUE/FALSE*

(d) For a loaded audio cable the attenuation/frequency response contains a region in which the attenuation is constant up to a point where a rapid reduction in attenuation occurs. *TRUE/FALSE*

(e) The relative group-delay/frequency response of a loaded audio cable shows a fall in group delay at high frequencies relative to the group delay at low frequencies. *TRUE/FALSE*
(*8 min*)

A2 [*Tutorial note:* See Q1 for actual curves (these 2 questions would not normally have been set in the same examination).]

(*a*) False
(*b*) False
(*c*) True
(*d*) False
(*e*) True

Q3 (a) List the 4 primary coefficients of a transmission line.

(b) Show how these primary coefficients can be arranged to represent a pair of wires.

(c) Explain briefly how these primary coefficients affect the frequency response of a cable.

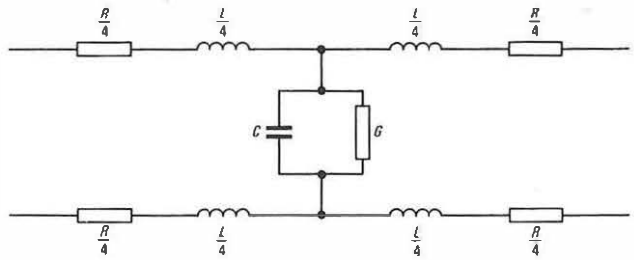
(d) Indicate whether the following statements are true or false:

(i) Distortion on a line will occur when the characteristic impedance of the line varies with frequency and the line is terminated in an impedance that does not vary in an identical manner. *TRUE/FALSE*

(ii) Distortion on a line will occur when the attenuation of the line varies with frequency, so that the waves of different frequencies are attenuated by equal amounts. *TRUE/FALSE*
(*10 min*)

A3 (*a*) The four primary coefficients of a transmission line are: resistance (*R*), capacitance (*C*), inductance (*L*), conductance (leakance) (*G*).

(*b*) The arrangement of the primary coefficients is shown in the sketch.



(*c*) In a uniform line the coefficients are equally distributed along the line.

The resistance, which is in series, will give a constant attenuation for a given length of cable.

The inductance, which is also in series, will present an increasing reactance with increasing frequency.

The capacitance, which is in parallel, will have a shunting effect; that is, the reactance will decrease with increasing frequency.

The conductance, which is the reciprocal of the insulation resistance, will be constant.

[*Tutorial note:* The values of the primary coefficients remain constant only at low frequencies.]

(*d*) (*i*) True

(*ii*) False. [*Tutorial note:* The statement should read, ". . . so that the waves of different frequencies are attenuated by *different* amounts."]

Q4 Give a brief description of the sources of group-delay/frequency distortion in a telecommunications system. (*6 min*)

A4 Group-delay/frequency distortion arises from non-linear phase/frequency characteristics of a network which arises from various sources of filtration in the transmission path. For example,

(*a*) inductively-loaded audio cables which act as low-pass filters with a cut-off frequency at about 4 kHz,

(*b*) channel-translation equipment includes band-pass filters which increase delay towards the upper and lower frequencies in the audio band,

(*c*) through-group filters distort channels 1 and 12.

Q5 Explain the effects of

(*a*) *attenuation/frequency distortion, and*

(*b*) *group-delay/frequency distortion*

on analogue and digital signals. (*10 min*)

A5 (*a*) All signals occupy certain natural bandwidths; the more complex the signal the wider the bandwidth. For the purposes of transmission it is often necessary to restrict this bandwidth but still retain intelligibility, and within this restricted bandwidth the frequency response should ideally be constant. However, signals passed along a cable suffer attenuation which increases with frequency. This can result in the higher frequencies being excessively reduced in level, causing poor quality music reproduction, poor definition in television pictures, poor speech reception and errors in telegraph reception. The recipient of an analogue signal is often the human ear or eye, which can interpret even poor quality signals with a good measure of success. This implies that relatively narrow bandwidths are acceptable for analogue transmission.

However, for the transmission of digital signals the receiving terminals are to a large extent intolerant of poor signals compared to that of a human and, as the rate at which binary digits (bit rate) can be successfully transmitted is proportional to bandwidth, the speed of transmission will be limited.

(*b*) The human ear is relatively insensitive to phase distortions of audio signals and, thus, group delay of analogue signals is not considered significant.

However, for digital networks the much wider bandwidths required and the consequent increase in group-delay distortion affect the maximum bit rate. The difference in the delay of different frequency components may cause interference between successive signalling states, resulting in an overlap of the frequencies representing the 0 and 1 states.

Q6 Indicate whether the following statements regarding the propagation of pulses in cables are true or false.

(*a*) *The time taken for the current to rise to approximately 63% of its final value or to fall to 37% of its initial value is called the time constant and is equal to 0·7CR seconds.* *TRUE/FALSE*

(*b*) *In practice, it is assumed that the current reaches its final value in approximately 5CR seconds.* *TRUE/FALSE*

(*c*) *The reversal of polarity in double-current systems opposes the discharge of line capacitance, making it possible to achieve higher signalling speeds than for single-current working.* *TRUE/FALSE*

(*d*) *The reliability of a receiving relay is improved when using double-current signals because the build up of residual magnetism is prevented.* *TRUE/FALSE*

(*e*) *The silent interval shown on the arrival curve occurs after the input pulse has ceased.* *TRUE/FALSE*

(*8 min*)

A6 (*a*) False. [*Tutorial note:* The time constant of a resistance/capacitance circuit is given by *CR* seconds.]

(*b*) True. [*Tutorial note:* The following table represents the percentage of the final value compared with the time constant.

Time interval	% of input level
0	0
CR	63·2
2*CR*	86·5
3*CR*	95·0
4*CR*	98·2
5*CR*	99·3
∞	100·0]

(*c*) False. [*Tutorial note:* The reversal of polarity assists the discharge of line capacitance.]

(*d*) True.

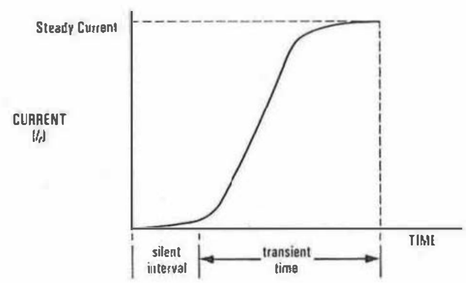
(*e*) False. [*Tutorial note:* For a long line there is a short interval after the leading edge of the input pulse (at the sending end) before the receive current is evident. This is called the silent interval.]

Q7 (*a*) *Sketch and label an arrival curve for a typical unloaded audio cable presented with a step function applied to the sending end.*

(*b*) *Briefly explain the main regions of the curve in part* (*a*).

(*c*) *Briefly explain the use of the arrival curve.* (*8 min*)

A7 (*a*)



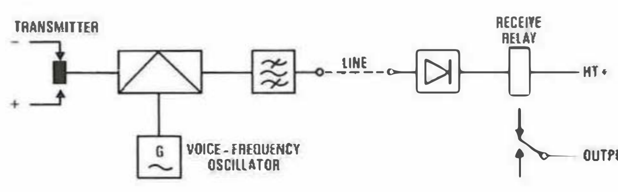
(*b*) The silent interval is the propagation time of the pulse; the transient is the time taken for the received current, I_r, to attain its steady-state value. [*Tutorial note:* Propagation time = $\sqrt{(LC)}$ km^{-1} and for a cable of length l km, the silent interval = $l\sqrt{(LC)}$ seconds.]

(*c*) The arrival curve shows the response of a line to a step function (first order differential) and, with this information, the time required for the receive current to reach the operate current of the receiver can be determined.

Q8 (*a*) *Sketch and label a basic block diagram for a voice-frequency signalling system.*

(*b*) *List the advantages of voice-frequency signalling.* (*7 min*)

A8 (*a*)



(*b*) (*i*) Voice-frequency signals can use standard speech amplifiers.

(*ii*) Voice-frequency signals can be transmitted over any distance with suitable amplification.

(*iii*) The same line plant can be employed as for the telephone system.

Q9 Indicate whether the following statements relating to a digital network are true or false.

(a) In a digital network, white noise is cumulative. TRUE/FALSE

(b) Higher bit rates are obtainable if the average noise power is reduced. TRUE/FALSE

(c) It is essential that a modem is employed on a digital network. TRUE/FALSE

(d) Wideband noise is removed by the regenerators. TRUE/FALSE

(e) Impulse noise is produced by thermal agitation in resistive components. TRUE/FALSE *(8 min)*

A9 *(a)* False. [*Tutorial note:* White noise is not cumulative because it is effectively removed by each regenerator.]

(b) True.

(c) False. [*Tutorial note:* A modem is required to interface a digital signal to an analogue system to overcome the problems associated with the narrow bandwidth characteristics of the analogue (telephone) network.]

(d) True.

(e) False. [*Tutorial note:* The main source of impulse noise is from switching circuits; for example, exchange switching equipment, teleprinters, electrical motors.]

Q10 From the following statements regarding pulse-code modulation (PCM) systems, select any that are false.

(a) PCM is a method of modulation used exclusively in frequency-division multiplex systems.

(b) PCM systems are essentially digital but the amplitude of the pulses sent to line is proportional to the amplitude of the information.

(c) PCM incorporates 3 essential processes; that is, pulse-amplitude modulation, quantisation and encoding.

(d) Coding of the sampled signal is achieved by using an encoder that compares the sample with a number of standard magnitudes.

(e) Quantisation noise is generated whenever an amplitude difference between an original sample and a reconstructed sample exists.

(f) The amplitude of the quantisation noise is independent of the size of the quantisation steps.

(g) Quantisation noise is reduced by using more quantum levels, but with the penalty of needing a greater number of binary codes.

(h) Each time slot in a 30-channel PCM system contains an 8-bit word. *(10 min)*

A10 Statements *(a)*, *(b)* and *(f)* are false.
[*Tutorial notes:*

(a) PCM is employed in time-division multiplex systems.

(b) The signals sent to line are of equal amplitude in the form of coded binary signals.

(f) Quantisation noise is proportional to the size of the quantisation steps.]

Q11 (a) Calculate the bit rate of a PCM system with the following parameters:

the signal is sampled at a rate of 8 kHz;
there are 24 channels; and
there are 256 binary codes.

(b) Calculate the bandwidth necessary to transmit this signal. Assume the signal has only 2 states and the system is noise free. *(10 min)*

A11 *(a)* Bit rate = sampling frequency × number of bits per word × number of channels,

$$= 8 \times 10^3 \times 8 \times 24,$$
$$= \underline{1 \cdot 536\ \text{Mbit/s}}.$$

[*Tutorial note:* 8 bits are required to represent 256 codes.]

(b) The channel capacity of a noise-free system (bit/s) equals $2 \times$ bandwidth $\times \log_2 N$ (where N is the number of states).

$$\therefore \quad 1 \cdot 536 \times 10^6 = 2 \times \text{bandwidth} \times \log_2 2.$$

$$\therefore \quad \text{bandwidth} = \frac{1 \cdot 536 \times 10^6}{2 \times \log_2 2},$$

$$= \underline{768\ \text{kHz}}.$$

[*Tutorial note:* In order to calculate $\log_2$, the definition of a logarithm should be remembered; that is, a logarithm of a number is the power to which the base must be raised to equal the number.

For a multistate system (more than 2 states), the bandwidth requirements will reduce.]

Q12 Explain briefly why only the first pair of side frequencies is required for the frequency modulation of digital signals. *(5 min)*

A12 Frequency modulation of complex waveforms results in numerous side frequencies but, for a digital signal that can be represented by two discreet frequencies, sufficient information will be contained in the first pair of side frequencies, which suggests that they are simply amplitude-modulated signals.

Q13 State the advantage of using only one pair of side frequencies for a frequency-shift keyed system. *(2 min)*

A13 The message can be transmitted using a restricted bandwidth.

Q14 State the 2 main requirements when carrier frequencies for frequency-shift keying are chosen. *(2 min)*

A14 *(a)* The bandwidth of the circuit, and

(b) the bit rate (or speed of data transmission).

Q15 Complete the following statements relating to frequency-shift keying (cross out the incorrect entry).

(a) The closer the frequencies are together the easier/more difficult it is to detect the difference between them.

(b) The faster/slower the bit rate, the closer the frequencies can be together. *(2 min)*

A15 The following should have been deleted:

(a) easier, and

(b) faster.

Q16 Explain the terms "dibit" and "tribit" coding in relation to phase modulation of a binary signal. *(6 min)*

A16 The data stream (string of binary digits) is split into groups of 2 bits for dibit coding and groups of 3 bits for tribit coding. The groups of 2 bits can represent 4 states which are implemented by 4 changes of phase of the carrier. Similarly, the 3-bit groups are represented by 8 changes in the phase of the carrier.

[*Tutorial note:* The following tables show the phase changes for dibit and tribit coding respectively.

dibit	phase change
00	0°
01	90°
10	180°
11	270°

tribit	phase change
000	45°
001	90°
010	135°
011	180°
100	225°
101	270°
110	315°
111	360°]

Q17 Select the correct answer from the following.
For a practical optical fibre, to enable the light to be totally internally reflected within the glass rod, it needs to be surrounded by

(a) a medium of equal refractive index,

(b) a medium of higher refractive index,

(c) a medium of lower refractive index, or

(d) air. *(3 min)*

A17 *(c)* [*Tutorial note:* Total internal reflection can occur only when light travels from one medium to another medium with a lower refractive index.]

Q18 State the main type of modulation technique used for optical-fibre transmission systems. *(2 min)*

A18 Direct modulation or digital modulation; that is, the light source is switched ON and OFF in sympathy with a binary code.

Q19 *The refractive index (μ) of a medium is given by*

(*a*) $\dfrac{\textit{sine of the incident ray}}{\textit{cosine of the reflected ray}}$,

(*b*) $\dfrac{\textit{sine of the reflected ray}}{\textit{sine of the incident ray}}$,

(*c*) $\dfrac{\textit{sine of the incident ray}}{\textit{sine of the reflected ray}}$, *or*

(*d*) $\dfrac{\textit{cosine of the incident ray}}{\textit{sine of the reflected ray}}$. (*2 min*)

A19 (*c*)

Q20 *Match the items in the left-hand column with the most appropriate in the right-hand column. All items refer to optical fibres.*

(*a*) *In monomode transmission*
(*b*) *In graded-index fibre*
(*c*) *In multimode transmission*
(*d*) *In stepped-index fibre*

(*A*) *the refractive index of fibre changes continuously from the centre to the boundary.*
(*B*) *only one ray of light is propagated along the fibre.*
(*C*) *there is a discreet step in the refractive index between core and cladding.*
(*D*) *more than one ray of light is propagated along the fibre.*
(*3 min*)

A20 (*a*) (*B*)
(*b*) (*A*)
(*c*) (*D*)
(*d*) (*C*)

Q21 *State the 2 factors that limit optical-fibre bandwidth.* (*2 min*)

A21 Multimode dispersion and material dispersion.

Q22 *Select two false statements from the following, all of which relate to light sources and detectors.*

(*a*) *A laser has a linear optical-power/drive-current characteristic.*
(*b*) *A light-emitting diode (LED) has a linear optical-power/drive-current characteristic.*
(*c*) *A laser can produce greater light output than an LED.*
(*d*) *An LED and laser operate in the ultra-violet wavelength range.*
(*4 min*)

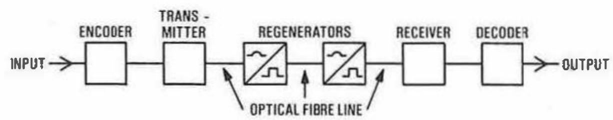
A22 (*a*) [*Tutorial note:* A laser is non-linear.]
(*d*) [*Tutorial note:* Both operate in the infra-red wavelength range.]

Q23 *State the range of wavelengths used for optical-fibre communication.* (*2 min*)

A23 800–1300 nm, which is within the infra-red band.

Q24 *Sketch and label a simple block diagram of a unidirectional optical-fibre transmission system.* (*5 min*)

A24



R. W. Kellingray

SCOTTISH TECHNICAL EDUCATION COUNCIL

Certificate in Electrical and Electronic Engineering

The questions given below are from examination papers set by the Scottish Technical Education Council (SCOTEC) and are reproduced with the permission of the SCOTEC. The answers given have been prepared by independent authors. Sometimes, additional tutorial information is given; this is enclosed within square brackets to distinguish it from the information that would be expected of students under examination conditions.

SCOTEC: MATHEMATICS III 1982

This examination paper is for the revised syllabus. Students were expected to answer all questions in section A and 4 questions from section B. The time allowed for the paper was 3 h. Students were instructed to give answers to numerical questions to 3 significant figures, unless otherwise stated. The use of calculators was permitted. The following list of formulae was included with the paper.

FORMULAE SHEET

ALGEBRA

$$\ln x = \log_e x = \frac{\log_{10} x}{\log_{10} e}$$

$$\exp x = e^x$$

Solutions of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

TRIGONOMETRY

$\sin^2 A + \cos^2 A = 1$ $\qquad \tan A = \dfrac{\sin A}{\cos A}$

CALCULUS

Note: Notations for first derivative of $y = f(x)$ are $\dfrac{dy}{dx}$, $f'(x)$, y', Dy

$\dfrac{d}{dx}(x^n) = nx^{n-1}$ $\qquad$ D (constant) = 0

$\dfrac{d}{dx}(\sin x) = \cos x$ $\qquad$ $D[kf(x)] = kf'(x)$, k constant

$\dfrac{d}{dx}(\cos x) = -\sin x$ $\qquad$ $D(u + v) = Du + Dv$

$\frac{d}{dx}(\exp x) = \exp x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Velocity and Acceleration

velocity $v = \frac{ds}{dt}$

acceleration $f = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$

COMBINATION LOGIC

$A + A.B = A$ $\quad$ $A.(A + B) = A$

$A + \bar{A} = 1$ $\quad$ $A.\bar{A} = 0 \quad \bar{\bar{A}} = A$

$\overline{A + B} = \bar{A}.\bar{B}$ $\quad$ $\overline{A.B} = \bar{A} + \bar{B}$

$A + B.C = (A + B).(A + C)$

SECTION A

Q1 Find in the form $r\angle\theta$ the sum of Z_1 and Z_2, where $Z_1 = 2 - j3$ and $Z_2 = 1 + j7$. (*6 marks*)

A2 $Z_1 + Z_2 = 3 + j4.$

$r = \sqrt{(3^2 + 4^2)} = 5.$

$\theta = \arctan\left(\frac{4}{3}\right) = 53\cdot13°.$

$Z_1 + Z_2 = \underline{5\angle 53\cdot13°}.$

Q2 Solve for x, the equation $2x^2 - x - 3 = 0$. (*5 marks*)

A2 $2x^2 - x - 3 = 0.$

$(2x - 3)(x + 1) = 0.$

$\therefore \quad \underline{x = \frac{3}{2} \text{ or } -1}.$

[*Tutorial Note:* Alternatively the formula

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a},$$

where $a = 2$, $b = -1$, and $c = -3$, can be used.]

Q3 Given that $2 \ln x + \ln 4x = 4\cdot68$, find the value of x to one significant figure. (*8 marks*)

A3 $2 \ln x + \ln 4x = 4\cdot68.$

$\therefore \quad \ln x^2 + \ln 4x = 4\cdot68.$

$\therefore \quad \ln 4x^3 = 4\cdot68.$

$\therefore \quad 4x^3 = e^{4\cdot68},$

$= 107\cdot8.$

$\therefore \quad x^3 = 26\cdot94.$

$\therefore \quad x = \underline{2\cdot998 = 3}$, to one significant figure.

Q4 The instantaneous current i amperes is given by $i = 10e^{\frac{-t}{CR}}$ when a capacitor is being charged. The capacitance $C = 8\cdot8 \times 10^{-6}$ and resistance $R = 1\cdot1 \times 10^6$. Find i when $t = 1\cdot5$. (*6 marks*)

A4 $\frac{t}{CR} = \frac{1\cdot5}{8\cdot8 \times 10^{-6} \times 1\cdot1 \times 10^6},$

$= \frac{1\cdot5}{9\cdot68 \times 10^0},$

$= 0\cdot155.$

$\therefore \quad i = 10e^{-0\cdot155},$

$= \underline{8\cdot56 \text{ A}}.$

Q5 Differentiate:

(*a*) $y = 2x^3 - \frac{1}{x} - 4$;

(*b*) $y = \sqrt{(2 - 3t)}$; *and*

(*c*) $y = 3 \cos 2\theta$. (*3, 4, 3 marks*)

A5 (*a*) $\underline{\frac{dy}{dx} = 6x^2 + \frac{2}{x^2}}.$

(*b*) $y = (2 - 3t)^{1/2}.$

$\frac{dy}{dt} = \frac{1}{2} \times (-3) \times (2 - 3t)^{-1/2},$

$= \underline{\frac{-3}{2\sqrt{(2 - 3t)}}}.$

(*c*) $\underline{\frac{dy}{d\theta} = -6 \sin 2\theta}.$

Q6 Simplify the logic function

$f = A.(\bar{A} + B) + B.(B + C) + B.$ (*5 marks*)

A6 $f = A.\bar{A} + A.B + B.B + B.C + B,$

$= A.B + B + B.C + B,$

$= B + B.(A + C),$

$= \underline{B}$, by the law of absorption.

Q7 Find the co-ordinates of the turning points on the curve whose equation is $y = x^3 - 6x^2 + 9x$ and determine their nature. (*10 marks*)

A7 At the turning points $\frac{dy}{dx} = 0.$

$\frac{dy}{dx} = 3x^2 - 12x + 9.$

$\frac{dy}{dx} = 0$ when,

$3x^2 - 12x + 9 = 0.$

$\therefore \quad x^2 - 4x + 3 = 0.$

$\therefore \quad (x - 1)(x - 3) = 0.$

$\therefore \quad x = 1$, or 3.

When $x = 1$, $y = 1 - 6 + 9 = 4$.

When $x = 3$, $y = 27 - 54 + 27 = 0$.

The turning points are $\underline{(1,4)}$ and $\underline{(3,0)}$.

Now, $\frac{d^2y}{d^2x} = 6x - 12.$

When $x = 1$, $\frac{d^2y}{dx^2} = -6.$

When $x = 3$, $\frac{d^2y}{dx^2} = +6.$

Hence, (1,4) is a local maximum, and (3,0) is a local minimum.

SECTION B

Q8 An alternating voltage V volts at time t seconds is given by

$V = 80 \sin (250\pi t + 0\cdot12)$.

(*a*) *State the maximum value of V and the frequency, and calculate the smallest positive value of t for which this maximum value of V occurs.* (*6 marks*)

(*b*) *Calculate the first positive value of t for which V is zero.* (*3 marks*)

(*c*) *Copy and complete the following table.* (*8 marks*)

t (*seconds*)	0	0·001	0·002	0·003	0·004	0·005	0·006	0·007	0·008
V (*volts*)	9·6		79		−9·6		−79		9·6

(*d*) *Plot the graph of V against t using these tabulated values.* (*8 marks*)

A8 (*a*) The maximum value of V occurs when $\sin(250\pi t + 0\cdot12) = 1$
Therefore, the maximum value of V is $\underline{80 \text{ V}}$.

The frequency, f, is given by

$$2\pi f = 250\pi.$$

$$\therefore \quad \underline{f = 125 \text{ Hz}.}$$

The first maximum occurs when

$$250\pi t + 0\cdot12 = \frac{\pi}{2}.$$

$$\therefore \quad 250\pi t = 1\cdot571 - 0\cdot12,$$

$$= 1\cdot451.$$

$$\therefore \quad t = \frac{1\cdot451}{250\pi},$$

$$= \underline{0\cdot00185 \text{ s} = 1\cdot85 \text{ ms}.}$$

(*b*) V is zero when $250\pi t + 0\cdot12 = 0, \pi, 2\pi, \ldots\ldots$

Since $250\pi t + 0\cdot12 = 0$ gives a negative value of t, the first positive value of t is given by

$$250\pi t + 0\cdot12 = \pi,$$

$$\therefore \quad 250\pi t = 3\cdot022.$$

$$\therefore \quad t = \frac{3\cdot022}{250\pi},$$

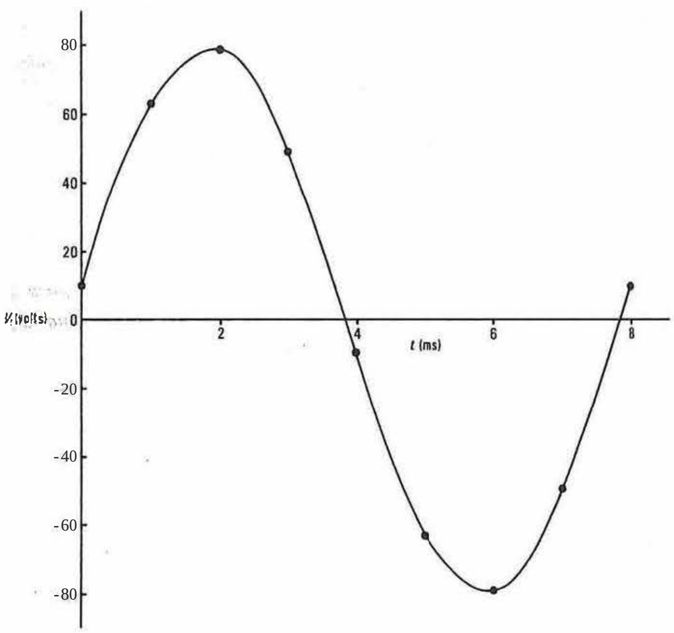
$$= \underline{0\cdot00385 \text{ s} = 3\cdot85 \text{ ms}.}$$

[*Tutorial Note:* The period is 8 ms. The maximum occurs at one-quarter period; that is, 2 ms after V = 0.]

(*c*)

t (seconds)	0	0·001	0·002	0·003	0·004	0·005	0·006	0·007	0·008
V (volts)	9·6	63	79	49	−9·6	−63	−79	−49	9·6

(*d*) See sketch.



Q9 The experimental values given in the following table are thought to obey a law of the type $y = ax^n$.

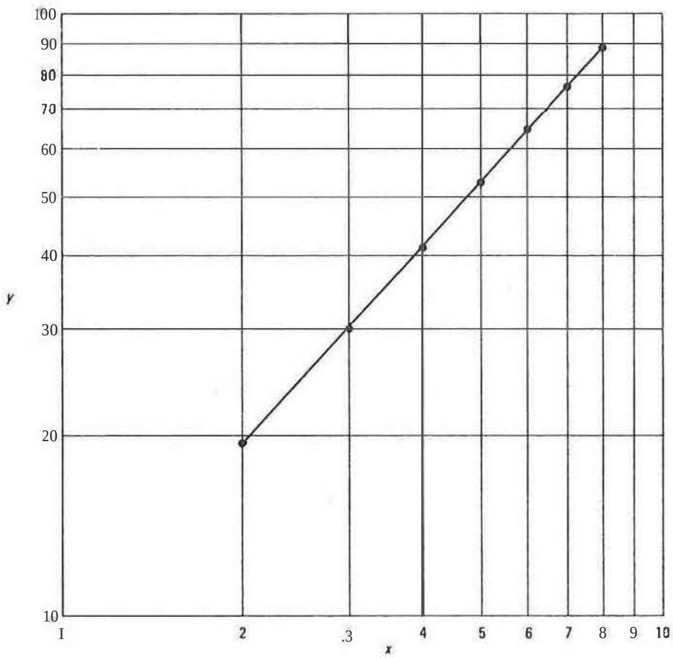
x	2	3	4	5	6	7	8
y	19·3	30	41·4	52·8	64·6	76·5	89

(*a*) *Plot the points on the log-log graph paper provided and verify this assumption.* (*12, 1 marks*)

(*b*) *Find the values of a and n.* (*8 marks*)

(*c*) *Using your graph, or otherwise, find the value of x when y = 60 and find the value of y when x = 7·5.* (*4 marks*)

A9 (*a*) The graph is shown in the sketch. Since the points lie on a straight line, the data satisfies the equation $y = ax^n$.



(*b*) The line passes through the points (2, 19·3) and (8, 89). Substituting these values for x and y in the equation $y = ax^n$,

$$19\cdot3 = a \times 2^n, \text{ and} \qquad \ldots\ldots (1)$$

$$89 = a \times 8^n. \qquad \ldots\ldots (2)$$

By taking logarithms of both equations,

$$\log 19\cdot3 = \log a + n \log 2, \text{ and} \qquad \ldots\ldots (3)$$

$$\log 89 = \log a + n \log 8. \qquad \ldots\ldots (4)$$

Subtracting equation (3) from equation (4),

$$\log 89 - \log 19\cdot3 = n \log 8 - n \log 2.$$

$$\therefore \quad \log \frac{89}{19\cdot3} = n \log \frac{8}{2}.$$

By taking logarithms to the base 10,

$$0\cdot6638 = 0\cdot6021n.$$

$$\therefore \quad \underline{n = 1\cdot10.}$$

Substituting in equation (1),

$$19\cdot3 = a \times 2^{1\cdot1},$$

$$= 2\cdot144a.$$

$$\therefore \quad a = \frac{19\cdot3}{2\cdot144},$$

$$= \underline{9\cdot00.}$$

(*c*) If $y = 60$,

$$60 = 9x^{1\cdot1}.$$

$$\therefore \quad x^{1\cdot1} = \frac{60}{9} = 6\cdot667.$$

$$\therefore \quad x = 6\cdot667^{1/1\cdot1} = \underline{5\cdot61.}$$

[*Tutorial Note:* The value of x can also be found from the graph.]

If $x = 7\cdot5$,

$$y = 9 \times 7\cdot5^{1\cdot1} = \underline{82\cdot6.}$$

Q10 (*a*) *The displacement S metres from a fixed point of a particle moving in a straight line is given by*

$$S = 3 + 9t + 3t^2 - t^3.$$

Obtain expressions for the velocity and acceleration of the particle and hence determine values of t for which

(*i*) *the velocity of the particle is zero,*

(*ii*) *the acceleration is zero.* (*4, 4, 2 marks*) (*b*) *Find the turning points on the graph of*

$$y = 2x^3 - 3x^2 - 12x + 4. \qquad (10\ marks)$$

(*c*) *Given* $P = \sin\theta + \cos\theta$, *find the smallest positive value of* θ *for which* $\frac{dP}{d\theta} = 0.$ *(5 marks)*

A10 (*a*) The velocity, V, is given by

$$V = \frac{dS}{dt} = 9 + 6t - 3t^2.$$

The acceleration, f, is given by

$$f = \frac{dV}{dt} = \underline{6 - 6t.}$$

(*i*) $V = 0$ when

$$9 + 6t - 3t^2 = 0.$$
$$\therefore \quad 3 + 2t - t^2 = 0.$$
$$\therefore \quad (3 - t)(1 + t) = 0.$$
$$\therefore \quad t = \underline{-1 \text{ or } 3.}$$

The result $t = -1$ is not practicable.

(*ii*) $f = 0$ when

$$6 - 6t = 0.$$
$$\therefore \quad \underline{t = 1.}$$

(*b*) At the turning points $\frac{dy}{dx} = 0.$

$$\frac{dy}{dx} = 6x^2 - 6x - 12,$$
$$= 6(x^2 - x - 2),$$
$$= 6(x - 2)(x + 1).$$

$\frac{dy}{dx} = 0$ when $x = 2$ or -1.

When $x = 2$, $y = 16 - 12 - 24 + 4 = -16$.

When $x = -1$, $y = -2 - 3 + 12 + 4 = 11$.

The turning points are $\underline{(2, -16) \text{ and } (-1, 11).}$

[*Tutorial Note:* Students were not asked to determine the nature of the turning points.]

(*c*)
$$\frac{dP}{d\theta} = \cos\theta - \sin\theta.$$

$\frac{dP}{d\theta} = 0$ when

$$\cos\theta - \sin\theta = 0.$$
$$\therefore \quad \theta = \underline{\frac{\pi}{4}.}$$

Q11 (*a*) *The currents* i_1 *and* i_2 *in a circuit are related by the following simultaneous equations.*

$$0{\cdot}5i_1 - 0{\cdot}4i_2 = -0{\cdot}7$$
$$1{\cdot}2i_1 - 0{\cdot}3i_2 = 3{\cdot}6$$

Find the values of i_1 *and* i_2. *(8 marks)*

(*b*) *Solve the equation*

$$7t^2 + 8t - 2 = 0.$$

Express the solution correct to 3 decimal places. *(7 marks)*

(*c*) (*i*) *Given* $\log_{10} x = 1 + \frac{3}{2}\log_{10} 4$. *Find x.* *(6 marks)*

(*ii*) *Evaluate* $\frac{\log 27 - \log 3.}{\log 81}$ *(4 marks)*

A11 (*a*)

$$0{\cdot}5i_1 - 0{\cdot}4i_2 = -0{\cdot}7. \qquad \ldots\ldots (1)$$
$$1{\cdot}2i_1 - 0{\cdot}3i_2 = 3{\cdot}6. \qquad \ldots\ldots (2)$$

Multiplying equation (1) by 3 and equation (2) by 4,

$$1{\cdot}5i_1 - 1{\cdot}2i_2 = -2{\cdot}1, \qquad \ldots\ldots (3)$$
$$4{\cdot}8i_1 - 1{\cdot}2i_2 = 14{\cdot}4. \qquad \ldots\ldots (4)$$

Subtracting equation (3) from equation (4),

$$3{\cdot}3i_1 = 16{\cdot}5.$$
$$\therefore \quad i_1 = \frac{16{\cdot}5}{3{\cdot}3} = 5.$$

Substituting for i_1 in equation (1),

$$0{\cdot}5 \times 5 - 0{\cdot}4i_2 = -0{\cdot}7.$$
$$\therefore \quad 0{\cdot}4i_2 = 3{\cdot}2.$$

Hence, the solution is $\underline{i_1 = 5, i_2 = 8.}$

(*b*) The solutions can be found from the equation given in the Formulae Sheet,

$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a},$$

where $a = 7$, $b = 8$, and $c = -2$.

$$\therefore \quad t = \frac{-8 \pm \sqrt{(8^2 - 4 \times 7 \times (-2))}}{2 \times 7}$$
$$= \frac{-8 \pm \sqrt{(64 + 56)}}{14},$$
$$= \frac{-8 \pm \sqrt{120}}{14},$$
$$= \frac{-8 \pm 10{\cdot}954}{14},$$
$$= \underline{0{\cdot}211 \text{ or } -1{\cdot}354.}$$

(*c*) (*i*)
$$\log_{10} x = 1 + \frac{3}{2}\log_{10} 4.$$
$$= \log_{10} 10 + \log_{10} 4^{3/2},$$
$$= \log_{10} 10 + \log_{10} 8,$$
$$= \log_{10} 80.$$
$$\therefore \quad x = 80.$$

(*ii*)
$$\frac{\log 27 - \log 3}{\log 81} = \frac{3\log 3 - \log 3}{4\log 3},$$
$$= \frac{2\log 3}{4\log 3},$$
$$= \underline{\tfrac{1}{2}.}$$

Q12 (*a*) *The equation* $\theta = a\cos t + b\sin t$ *gives the angular displacement of a point in an oscillating mechanism. Find expressions for the angular velocity and acceleration of the point.*

Given that the angular velocity is 24 rad/s when $t = 0$, *and the angular acceleration is 10 rad/s²* *when* $t = \pi$ *seconds, find the values of a and b.* *(10 marks)*

(*b*) *A rectangular block of metal of square cross section with side x cm, has a length L cm and a total surface area of 200 cm².*

(*i*) *Show that the length L is given by*

$$L = \frac{100 - x^2}{2x}. \qquad (5\ marks)$$

(*ii*) *Hence obtain the formula for the volume V in terms of x.* *(2 marks)*

(*iii*) *Find the maximum volume of the block.* *(8 marks)*

A12 (*a*) The angular velocity, ω, is given by

$$\omega = \frac{d\theta}{dt} = \underline{-a\sin t + b\cos t.} \qquad \ldots\ldots (1)$$

The angular acceleration, α, is given by

$$\alpha = \frac{d\omega}{dt} = -a\cos t - b\sin t. \qquad \ldots\ldots (2)$$

When $t = 0$, $\omega = 24$ rad/s. Substituting these values in equation (1),

$$24 = -a \times 0 + b \times 1.$$
$$\therefore \quad b = \underline{24.}$$

When $t = \pi$, $\alpha = 10$. Substituting these values in equation (2),

$$10 = -a \times (-1) - b \times 0.$$
$$\therefore \quad a = \underline{10.}$$

(*b*) (*i*) The surface area, A, of the block is given by

$$A = 2x^2 + 4xL.$$

But, $A = 200$ cm².

$$\therefore \quad 2x^2 + 4xL = 200$$
$$\therefore \quad 4xL = 200 - 2x^2.$$
$$\therefore \quad L = \frac{100 - x^2}{2x}.$$

(*ii*)
$$V = x^2L.$$

Substituting for L,

$$V = \frac{x(100 - x^2)}{2},$$

$$= 50x - \tfrac{1}{2}x^3.$$

(*iii*) The volume of the block is a maximum for the value of x which satisfies $\frac{dV}{dx} = 0$.

$$\frac{dV}{dx} = 50 - \frac{3x^2}{2},$$

$$= \frac{100 - 3x^2}{2}.$$

$\frac{dV}{dx} = 0$ when $100 - 3x^2 = 0$

$$\therefore \quad x^2 = \frac{100}{3}.$$

$$\therefore \quad x = \frac{10}{\sqrt{3}}.$$

At this point, dV/dx changes sign from positive to negative; therefore, this is a maximum turning point.

Substituting for x in the equation for the volume, the maximum volume

$$= \frac{\frac{10}{\sqrt{3}}\left(100 - \frac{100}{3}\right)}{2},$$

$$= \frac{5}{\sqrt{3}} \times \left(\frac{300 - 100}{3}\right),$$

$$= \frac{1000}{3\sqrt{3}}\ \text{cm}^2 = 192\ \text{cm}^2.$$

Q13 (*a*) *Simplify the logic function* $f = \overline{A.\bar{B} + A.\bar{C}} + \bar{A}$. (*8 marks*)

(*b*) *Show that* $\overline{A.B + \bar{A}.B + A.\bar{B}} = \bar{A}.\bar{B}$ *by drawing 2 Venn diagrams.* (*9 marks*)

(*c*) *From the truth table, express f as a logic function.* (*4 marks*)

A	B	$\bar{A}$	$\bar{B}$	f
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

(*d*) *Write down the truth table for the function* $x + \bar{y}$. (*4 marks*)

A13 (*a*)

$$f = \overline{A.\bar{B}}.\ \overline{A.\bar{C}} + \bar{A},$$

$$= (\bar{A} + \bar{\bar{B}}).(\bar{A} + \bar{\bar{C}}) + \bar{A},$$

$$= (\bar{A} + B).(\bar{A} + C) + \bar{A},$$

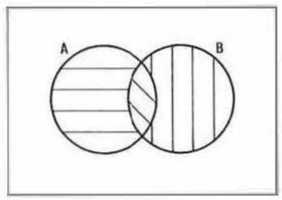
$$= \bar{A} + B.C + \bar{A},\ \text{(second distributive law)}$$

$$= \bar{A} + B.C.$$

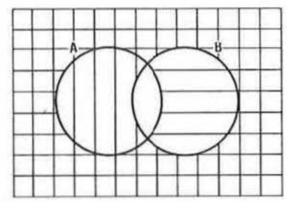
(*b*) In sketch (*a*), $A.B$ is shaded diagonally, $\bar{A}.B$ is shaded vertically, and $A.\bar{B}$ is shaded horizontally. Therefore, $A.B + \bar{A}.B + A.\bar{B}$ is the total shaded area, and $\overline{A.B + \bar{A}.B + A.\bar{B}}$ is the unshaded area.

In sketch (*b*) $\bar{A}$ is shaded horizontally, $\bar{B}$ is shaded vertically. Therefore, $\bar{A}.\bar{B}$ is the doubly-shaded area.

The unshaded area in sketch (*a*) is the same as the doubly-shaded area in sketch (*b*).



(*a*)



(*b*)

(*c*) $f = \bar{A}.\bar{B} + A.B.$

(*d*)

x	y	$\bar{y}$	$x + \bar{y}$
0	0	1	1
0	1	0	0
1	0	1	1
1	1	0	1

Answers contributed by M. Keates

Published by *British Telecommunications Engineering Journal*, 2–12 Gresham Street, London EC2V 7AG and printed in Great Britain by Unwin Brothers Limited, Old Woking, Surrey.