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P.O. ENGINEERING DEPARTMENT **EDUCATIONAL PAMPHLET - DRAFT SERIES**
ELECTRONICS 2/7

PUSH-PULL AMPLIFIERS

CONTENTS

INTRODUCTION

If a greater undistorted power output is required than that given by a single valve it is possible to use a larger valve or to connect two or more valves in parallel. A better alternative, however, is to connect a pair of valves in the manner shown in Fig. 1. The input voltage is divided into equal parts so that the voltages applied to the grids of the valves are equal in magnitude but 180° out of phase. As a result, the anode current of one valve is rising as that of the other valve is falling. Valves connected as shown in Fig. 1 are said to be operated in push-pull.

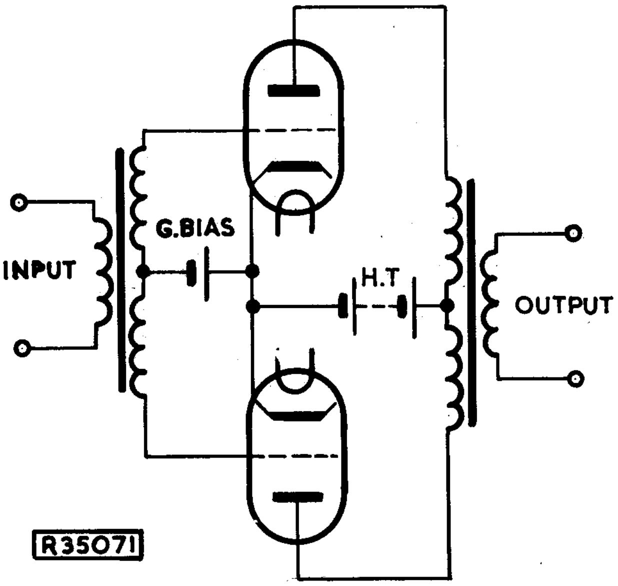


Fig. 1

There are several advantages in connecting valves in push-pull. Even order harmonic distortion, inherent in a single valve is cancelled out and the problem of d.c. saturation of the output transformer eliminated. This permits output powers greater than double the output power of a single valve to be obtained for the same percentage distortion. Alternatively, the same power output per valve can be obtained with a reduced distortion.

Push-pull amplifiers may be operated under either Class A or Class B conditions. Each method of working has its own particular advantages and disadvantages and are discussed in the pamphlet.

PUSH-PULL AMPLIFIERS WITH CLASS A BIAS

A Class A push-pull circuit with transformer coupling to the previous (driver) stage is shown in Fig. 2.

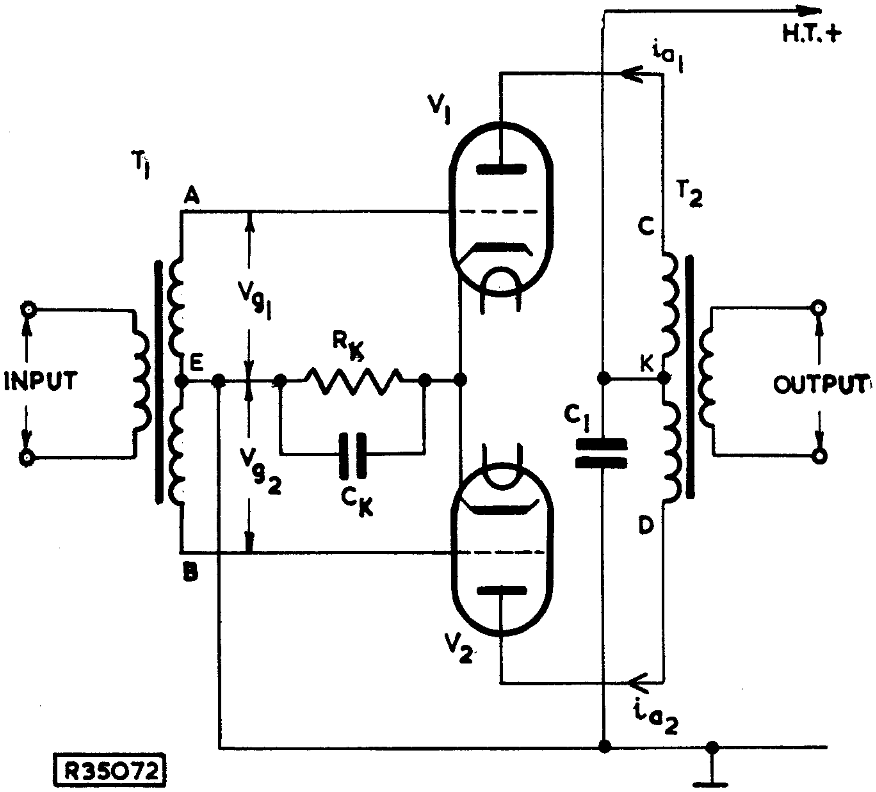


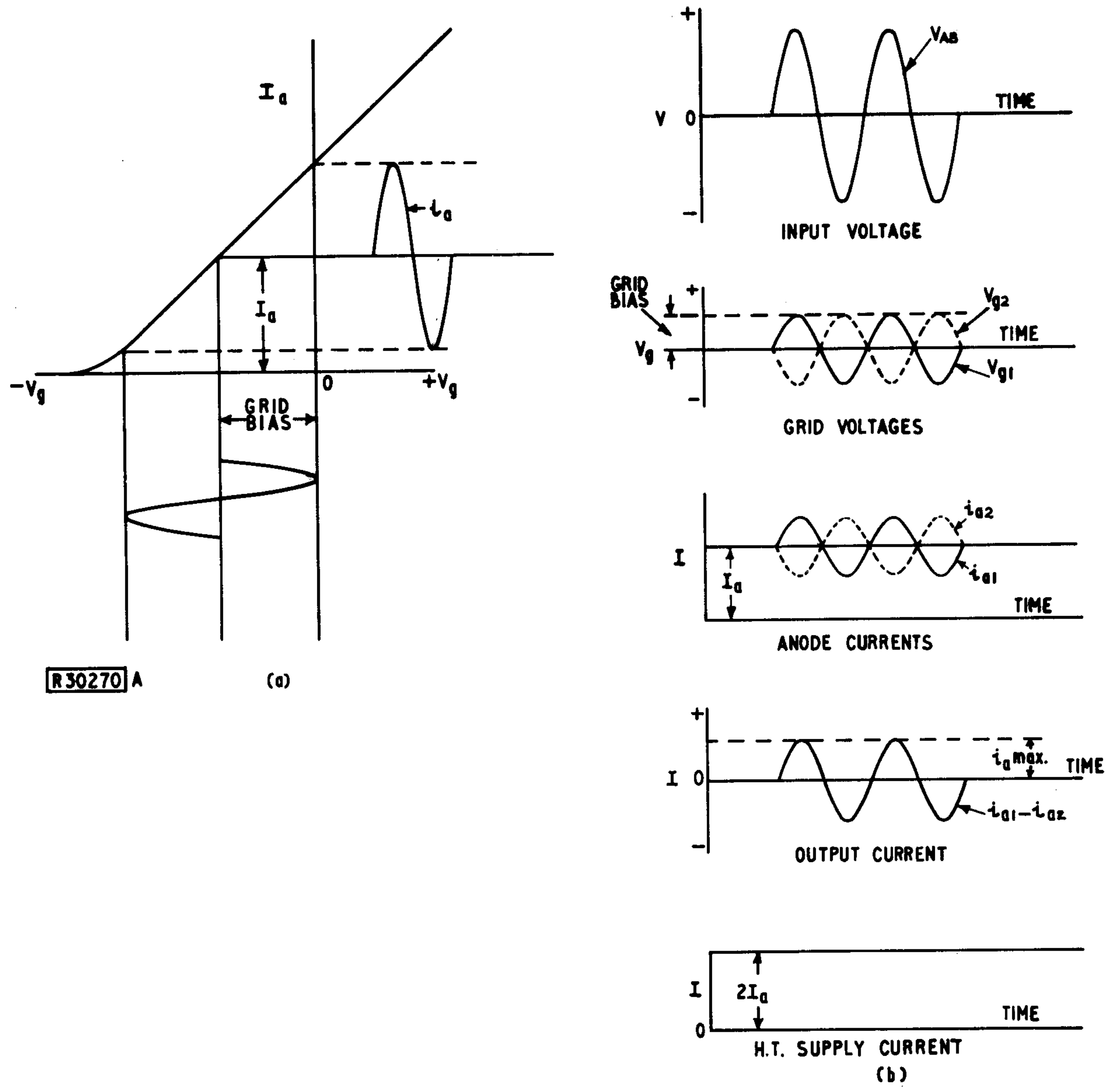
Fig. 2

The secondary windings of the input transformer T1 are connected to the grids of the two valves, the electrical mid-point, E, being common to the cathodes of both valves. Point E is also connected to h.t. negative, thus Rk is the common cathode resistor and is decoupled by capacitor Ck. The h.t. supply is connected between the electrical mid-point of the output transformer primary and the common cathode connexion.

The audio-frequency voltage is produced across AB by the action of the preceding stages of amplification. Whenever A is at a positive potential with respect to E, B is at an equal negative potential, and vice versa; i.e. the alternating p.d.s. from A to E and from B to E are always equal and opposite. Hence v_{g1} and v_{g2} are equal in amplitude, but 180° out of phase.

Now consider the output circuit: when no signal is being received a steady current I_a is flowing from anode to cathode of each valve, its value depending on the steady anode and grid voltages. The currents in the two valves are equal if the valves are perfectly matched and the h.t. supply positive connexion is at the electrical mid-point of the output primary. As regards the primary of the output transformer, however, these two equal currents flow in opposite directions through the two halves of the coil. The steady fluxes they produce in the coil are thus equal and opposite, and there is no resultant flux and magnetization of the core. Distortion of the waveform by partial saturation of the core, such as is likely to occur in the output transformer of a single valve output stage owing to the d.c. component of anode current, is thus prevented.

The best grid bias for each valve, for distortionless amplification, is the voltage half-way between the lower bend of the dynamic characteristic and the voltage at which grid current starts to flow, as shown in Fig. 3(a).



(a)

(b)

Fig. 3

The input voltage, V_{AB}, is shown in Fig. 3(b), and also the two equal voltages, v_{g1} and v_{g2}, which are applied to the grids of the respective valves. These grid voltages cause currents i_{a1} and i_{a2} to flow in phase with the respective grid voltages. These currents, which are 180° out of phase, subtract algebraicly in the output transformer to give an alternating current having a peak value of twice the alternating component of i_{a1} or i_{a2} taken separately.

Alternatively, consider v_{g1} to be decreasing i.e. tending to become more negative, and v_{g2} to be increasing simultaneously and at the same rate. Then the decreasing current, i_{a1}, in phase with v_{g1}, will induce the same voltage into the output transformer as the increasing current, i_{a2}. By suitable connexion of the windings CK and DK these voltages can be made additive, and twice the voltage obtained compared with the voltage due to one valve alone.

As i_{a1} decreases at the same rate as i_{a2} increases, the current taken from the supply is constant as shown in Fig. 3(b).

Fig. 2 shows a Class A push-pull circuit in which the grid bias voltage is obtained from a common cathode resistor, R_K, bypassed by capacitor C_K. In many circuits C_K is omitted as this provides a combination of negative and positive feedback which tends to equalize the individual anode currents of the two valves when the two halves of the circuit are not exactly balanced. It has however the disadvantage of increasing the odd harmonic distortion. This will not be further considered as feedback is outside the scope of this pamphlet.

The use of separate cathode resistors instead of a common cathode resistor is also quite common. It enables a more accurate d.c. balance to be obtained between the two halves of the circuit, because the resistance of each resistor can be altered until the steady anode currents of the valves are equal. It is necessary to decouple each cathode resistor in order to prevent current negative-feedback being applied over each half of the circuit. Current negative-feedback is undesirable as it will increase the output impedance of the amplifier, and, in general, a low output impedance is desirable for a power amplifier. The use of separate cathode resistors has the disadvantage that the self-balancing effect of the undecoupled common cathode resistor circuit is lost.

Harmonic distortion

One of the more important advantages of push-pull working is the elimination from the output of any second (and other even order) harmonic components, generated by the valves. The fundamental and the odd harmonics cancel in the h.t. supply and add in the secondary winding of the output transformer, whilst the even harmonics add in the h.t. supply but cancel in the output transformer secondary winding.

If the output waveforms of the two valves are drawn in the correct phase, as in Fig. 4, and added, it can be seen that while the second harmonics cancel out, the fundamental and third harmonics are additive. Thus the percentage third harmonic distortion is unaltered by push-pull operation. This is shown mathematically in Appendix 'A'.

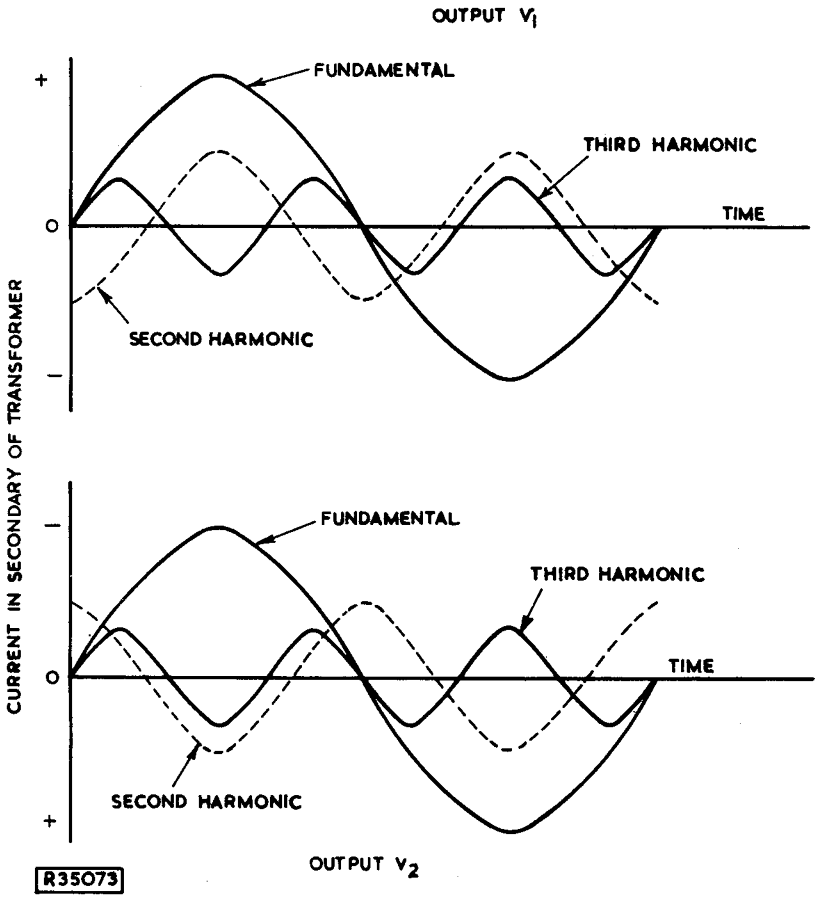


Fig. 4

DESIGN OF CLASS A PUSH-PULL AMPLIFIERS

When designing stages for operation in push-pull it is usual to work with what are known as "composite" characteristics for the pair of valves.

Composite characteristics

A set of characteristics can be drawn for a fictitious valve equivalent to two valves working in push-pull, the stage may then be designed by drawing a load line on these characteristics.

The following points must be taken into account when drawing these characteristics.

(a) Owing to the coupling between the two primary windings of the output transformer an increase in the anode voltage of one valve causes a corresponding decrease in the anode voltage of the other.

(b) The sense of the primary windings of the output transformer are such that the effective current in the primary is the difference between the currents in the two valves.

(c) The a.c. grid voltage is equal and opposite on each valve.

(d) The valve characteristics pass through their quiescent points together.

The composite curves may be drawn as follows:-

(1) Plot the anode characteristics of one valve and determine the operating point as for a single valve amplifier.

(2) Plot a second set of curves rotated through 180° and adjusted horizontally so that the steady h.t. voltages coincide, the operating points on the two sets of curves should then lie on the same vertical line. This operation takes into account points (a) and (d).

(3) Combine the two curves which pass through the operating points to give a single composite curve, equal at all points to the difference between the corresponding points on the two static curves, for each value of anode voltage. This operation takes into account point (b).

(4) Plot similar composite curves taking the difference in anode current between pairs of the static curves for values of bias equal amounts above and below the working bias. This operation takes into account points (b) and (c).

Fig. 5 shows the anode current/anode voltage characteristic of a typical triode valve.

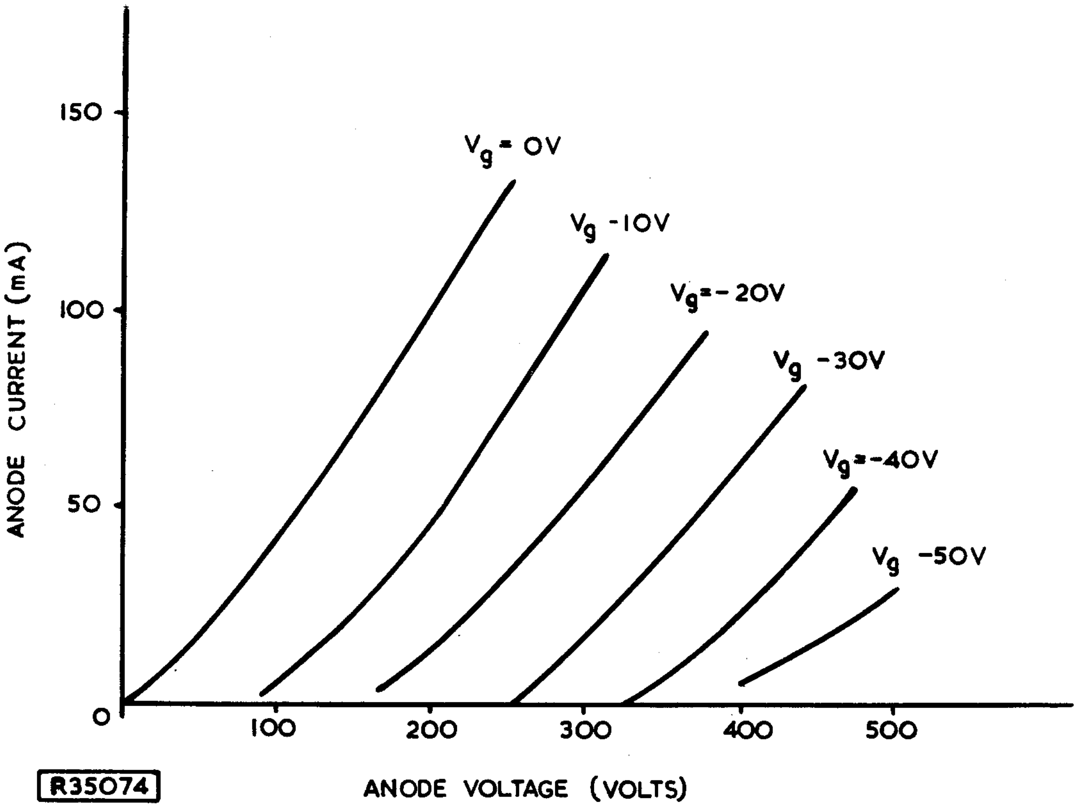


Fig. 5

If a pair of these triodes are operated in push-pull, composite characteristics may be drawn provided the h.t. and grid bias voltages are known. The composite characteristics shown in Fig. 6 are drawn on the assumption that the h.t. voltage is 250 volts and the grid bias voltage -20 volts.

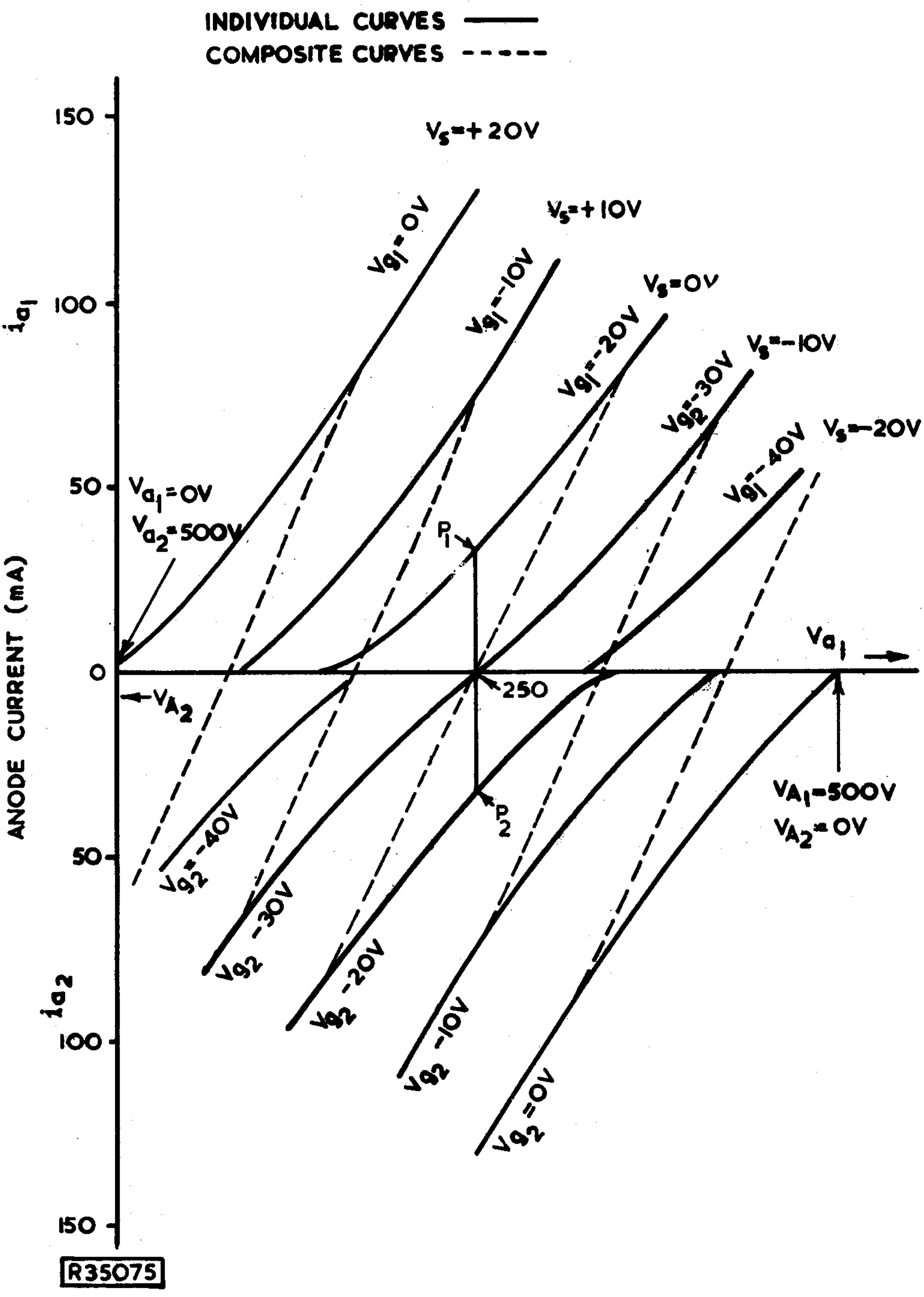


Fig. 6

The anode characteristics of one valve are rotated through 180° and drawn below the characteristics of the other valve, and adjusted horizontally until the h.t. voltages coincide. The operating points, P_1 and P_2, then lie on the same vertical line. The operating points are the points of intersection of a vertical line drawn from the 250 volt point, with the $v_{g_1} = -20$ volts and $v_{g_2} = -20$ volts characteristics. These characteristics, which pass through the operating points, are then combined to give a single composite curve, equal at all points to the difference between the corresponding points on the two static curves for each value of h.t. voltage, i.e. when the anode voltage of each valve is 250 volts, $i_{a_1} = i_{a_2}$ and thus the difference between the static curves, $i_{a_1} - i_{a_2}$, is zero.

Similarly, at the point Va_1 = 150 volts, i_{a1} = 0 mA, i_{a2} = 80 mA and $i_{a2} - i_{a1}$ = 80 mA, giving a second point on the composite curve.

Other pairs of individual characteristics are now combined to give composite characteristics for values of grid voltage equal amounts above and below the grid bias voltage of -20 volts.

As an example, consider a signal of 10 volts peak value applied to the push-pull stage. During one half-cycle the peak voltage applied to the grid of V_1 will be equal to -10 volts (-20 + 10) and that applied to the grid of V_2 -30 volts (-20 - 10). Thus a composite characteristic may be drawn for the static curves v_{g1} = -10 and v_{g2} = -30 volts in the same way as described above.

It must be noted the composite characteristics have ordinates representing the combined current changes of the two valves whilst their abscissae are the voltage changes of one valve only. Thus the slopes of the composite characteristics are twice that of the characteristics of either valve, hence the anode slope resistance of the composite valve is $\frac{r_a}{2}$ where r_a is the anode slope resistance of one valve.

Several features of the composite characteristics can be seen from inspection of Fig. 6.

(a) the quiescent anode current is zero,

(b) the composite characteristics are straighter than the individual valve characteristics,

(c) the anode slope resistance of the composite valve is reasonably constant over the range shown.

Composite load lines

It is shown in Appendix B that the composite valve works into an effective load of $\left(\frac{n}{2}\right)^2 R_s = \frac{n^2 R_s}{4}$ where the turns ratio n : 1 of the output transformer refers to the whole of the primary winding and R_s is the load connected to the secondary winding terminals.

The load presented by the transformer between points C to D (Fig. 2) is $n^2 R_s = R_L$. Thus the load on the composite valve is $\frac{R_L}{4}$ and therefore a composite load line may be drawn on the composite characteristics having a slope of $-\frac{4}{R_L}$ amps/volt as shown in Fig. 7.

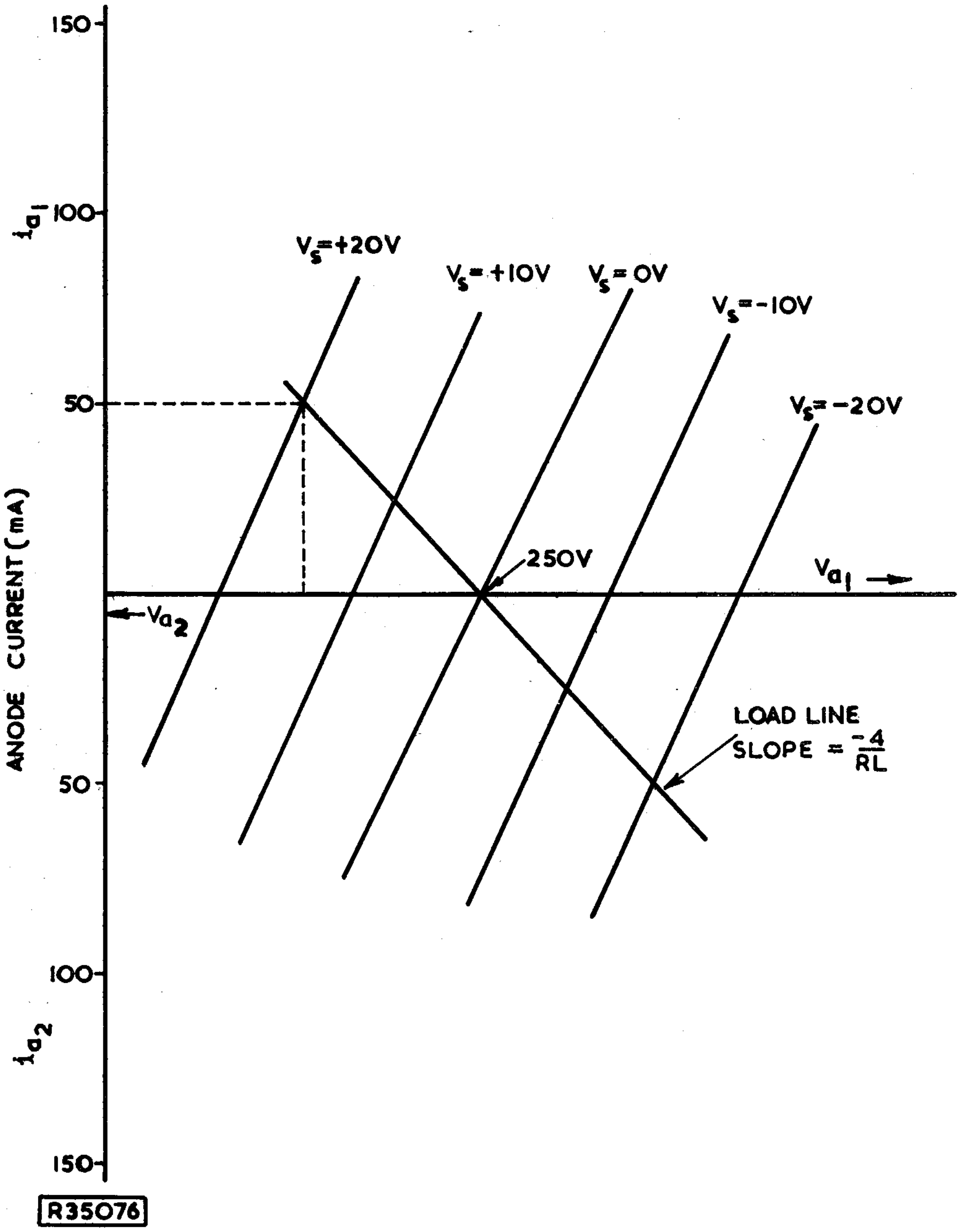


Fig. 7

The composite load line passes through the point 0 mA, 250 volts since, with no signal applied to the stage, the anode current of the composite valve is zero.

If the load between points C and D is not known, an optimum value can be found by drawing a number of load lines of varying slope and calculating the power output and distortion introduced for each load value.

Optimum Load

Due to the elimination of the second harmonic, the conditions governing maximum undistorted power output with single valve triode amplifiers no longer apply.

Maximum power output is obtained with triodes in push-pull when the composite load resistance, $\frac{R_L}{4}$, is equal to the anode slope resistance, $\frac{r_a}{2}$, of the composite valve.

$$\text{i.e. } \frac{R_L}{4} = \frac{r_a}{2}$$

$$\text{or } R_L = 2r_a$$

Thus for maximum power output the effective load between points C and D (Fig. 2) should be made equal to twice the anode slope resistance of either valve.

The composite load line may be utilized to determine the peak output current, power output etc. in a similar manner to that employed for a single valve amplifier. e.g. If the peak value of the input signal is 20 volts, then, from Fig. 7, the peak anode current is 50 mA.

Output waveforms

The composite load line may be used to plot the waveform of the output current in a similar manner to that employed for a single valve amplifier.

Consider a signal of peak value 20 volts applied to the push-pull valves, then V_s will vary between +20 and -20 volts (the voltage applied between grid and cathode of either valve varies between 0 and -40 volts). From the points of intersection of the corresponding characteristics with the composite load line it can be seen from Fig. 7 that the output current will vary between ± 50 mA.

If the waveform of the anode current of one of the valves is required the individual load line for that valve must be used. The individual load lines are plotted by drawing vertical lines through the intercepts of the composite curves and the composite load line to meet the corresponding individual characteristics and drawing a smooth curve through the points of intersection. Fig. 8 shows the individual load lines drawn in this manner.

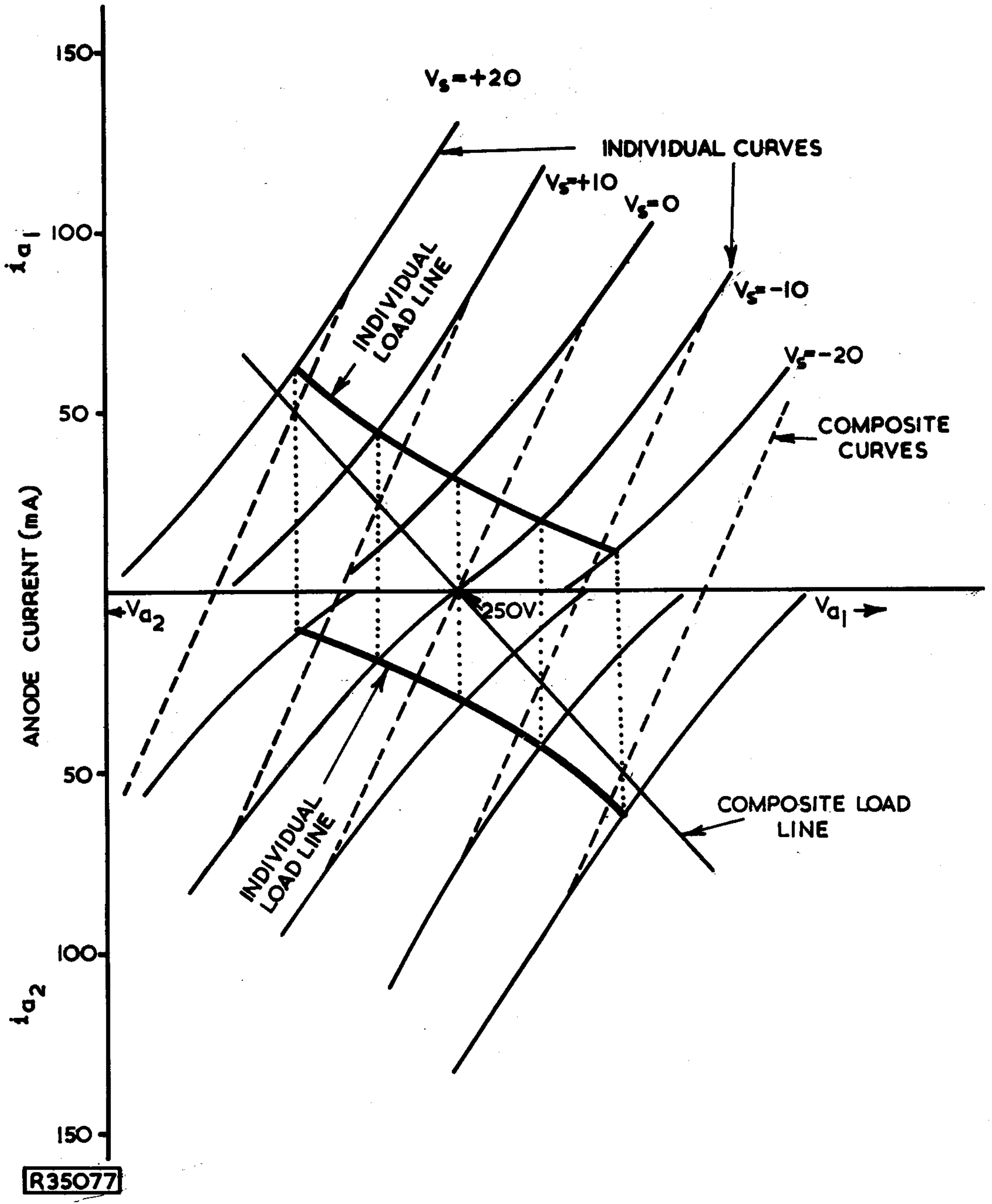


Fig. 8

The waveform of the anode current of one valve of the push-pull pair is shown in Fig. 9. An input signal of peak value 20 volts has been applied to the individual load line and the corresponding values of anode current plotted.

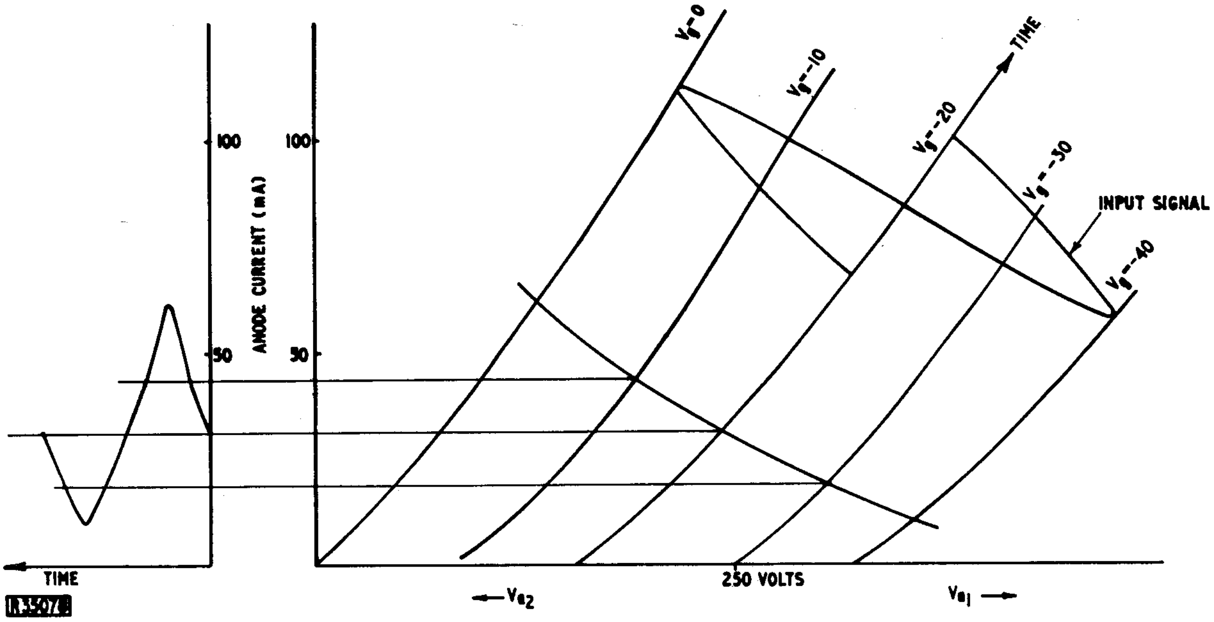


Fig. 9

It can be clearly seen from Fig. 9 that the anode current waveform of either valve taken separately is distorted.

Pentodes and beam tetrodes in Class A push-pull

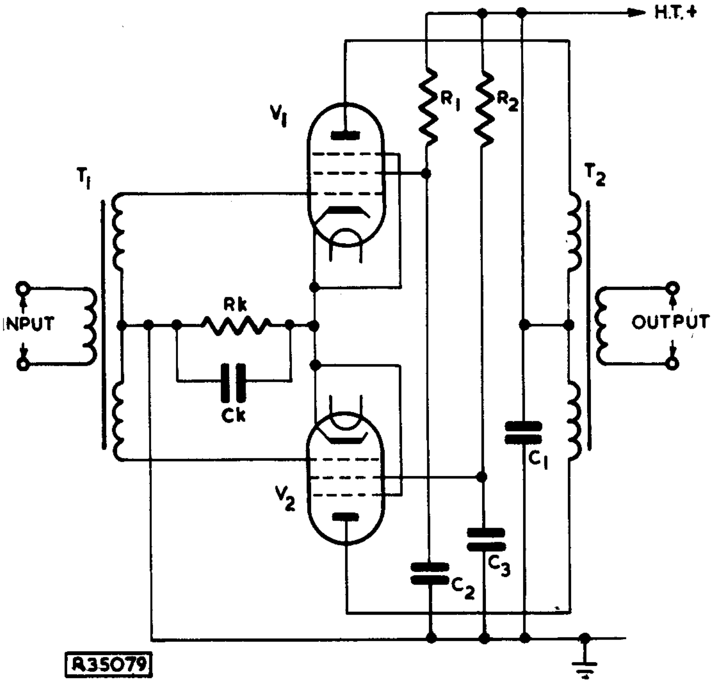


Fig. 10

The circuit of a push-pull amplifier employing pentode valves is shown in Fig. 10. The functions of the components are the same as those of the triode push-pull amplifier shown in Fig. 2, except for resistors R_1 and R_2 and capacitors C_2 and C_3 which are additional and form conventional screen grid dropper and decoupling circuits.

When pentodes (or beam tetrodes) are operated in push-pull the anode load of each valve should be chosen such that, if it were used with a single valve there would be mainly second harmonic and a minimum of third harmonic distortion. This is because second harmonics are eliminated by the push-pull connexion and third harmonics are not.

Since the third harmonic content of pentodes is considerably greater than that of triodes, the percentage distortion of a push-pull stage using pentodes is greater than that of a comparable triode stage. The power output with pentodes is greater than with triodes due to the higher gain of the pentode valve.

Due to its greater distortion a push-pull stage using pentodes as shown in Fig. 10 compares unfavourably with the triode circuit unless power output regardless of distortion (within limits) is the main consideration.

The distortion of a pentode circuit can be greatly reduced by the application of negative-feedback to the circuit, but this will not be discussed as it is outside the scope of this pamphlet. Another method of using pentodes (or tetrodes) in push-pull to obtain low values of distortion is shown in Fig. 11. The circuit is similar to the push-pull circuit shown in Fig. 10 but has the screen grids connected to tappings on the primary winding of the output transformer. The load presented by the output transformer between points A and D is R_L and the load between points B and C is kR_L, where k is a fraction depending on the position of the tapping points. When connected in this manner the output valves are said to be operated under 'Distributed Load Conditions'. Distributed load conditions combine the high gain of pentodes with the low distortion of triodes.

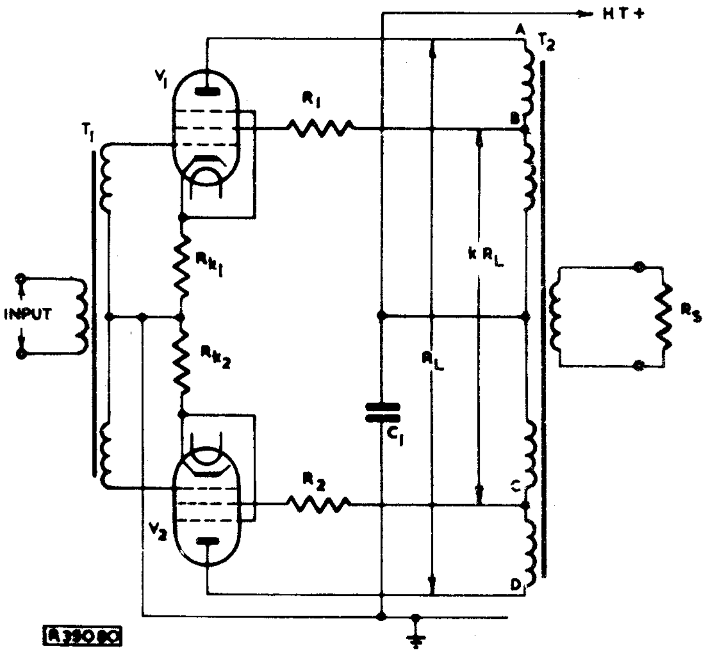


Fig. 11

Distributed Load Conditions

In the simplest form of distributed load, the screen grids of the output valves are fed from suitably positioned tapping points on the primary of the output transformer. The characteristics of a distributed load stage are intermediate between those for pentode and triode operation, approaching triode operation as the percentage of the primary winding common to the anode and screen grid circuits increases. It is found that about two-thirds of the power output of the corresponding pentode stage can be obtained with greatly reduced distortion, whilst with a power output corresponding to triode operation a similar degree of distortion obtains.

Table 1 gives some figures for a typical pentode power output valve.

TABLE 1

Operation	Percentage total distortion		
	at 5W	at 10W	at 15W
strapped as a triode	1.0	-	-
distributed load: 20% common winding	0.8	1.0	1.5
distributed load: 43% common winding	0.7	0.9	1.3
pentode connection	1.5	2.0	2.0

It can be seen from Table 1 that distributed load operation enables the power handling capacity to be double that possible with triode operation for the same degree of distortion. It would appear that there is little advantage to be gained by increasing by the percentage of common winding from 20% to 43%. However, as with triode operation, power output and percentage distortion are less dependent upon the value of load impedance for 43% common winding, than for 20%.

EFFICIENCY OF CLASS A PUSH-PULL AMPLIFIERS

The efficiency of a push-pull amplifier using triode valves and working under Class A conditions is given by:-

$$\text{efficiency} = \frac{\text{maximum power output}}{\text{d.c. power input}} \times 100\%$$

It is shown in E.P. DRAFT SERIES ELECTRONICS 2/2 that the maximum power output of a single triode power amplifier is:-

$$\text{Power output} = \frac{(E_{max} - E_{min})\ (I_{max} - I_{min})}{8} \text{ watts}$$

where $\frac{(E_{max} - E_{min})}{2}$ is the peak value of the alternating voltage developed across the primary of the output transformer

and $\frac{(I_{max} - I_{min})}{2}$ is the peak value of the alternating component of the anode current.

The maximum value of $(E_{max} - E_{min})$ is $2V_a$ and that of $(I_{max} - I_{min})$ is $2I_a$.

where V_a is the h.t. voltage (and quiescent anode voltage)

and I_a is the standing anode current of either valve.

$$\text{Maximum power output of either valve} = \frac{4\, I_a V_a}{8} \text{ watts}$$

$$= \frac{I_a V_a}{2} \text{ watts}$$

Hence the power output of the complete stage is given by:-

$$\text{Power output} = \frac{2\, I_a V_a}{2} \text{ watts}$$

$$= I_a V_a \text{ watts}$$

The d.c. input to the stage is $I_a V_a$ watts per valve.

$$\therefore \text{ d.c. supplied to the stage} = 2\, I_a V_a \text{ watts}$$

$$\text{Thus the maximum efficiency} = \frac{I_a V_a}{2\, I_a V_a} \times 100\%$$

$$= 50\%$$

In practice, $(I_{max} - I_{min})$ cannot approach the value $2\, I_a$ without the introduction of serious distortion. Thus the actual efficiencies of push-pull output stages are generally much lower than the theoretical maximum of 50%, typical efficiencies being 20 - 25%.

When the stage employs pentode or beam tetrode valves the practical efficiency is even lower because of the extra power dissipated at the screen grid.

Example

Two similar pentodes operate in Class A push-pull. The standing anode current of each is 77 mA. When a sinusoidal input signal is applied to the stage the peak fundamental, second and third harmonic currents are 35 mA, 3 mA and 2 mA respectively.

Calculate (a) the r.m.s. value of the total h.t. current

(b) the percentage of each harmonic in the output.

The fundamental and third harmonic components of the anode currents cancel in the h.t. supply and add in the secondary winding of the output transformer. The d.c. and second harmonic components add in the h.t. supply and the second harmonic component cancel in the output transformer.

(a) The d.c. supplied to each valve is 77 mA, hence the d.c. flowing in the h.t. supply is 77 x 2 = 154 mA. The h.t. current at the fundamental frequency = 0 mA. The h.t. current at the frequency of the second harmonic is 3 mA per valve or 6 mA total and the third harmonic component of the h.t. current is zero.

$$\therefore \text{ r.m.s. value of the total h.t. current} = \sqrt{154^2 + 6^2}$$

$$= \underline{154.12 \text{ mA}}$$

Answer

(b) The current in the secondary winding of the output transformer consists of fundamental and third harmonic only.

$\therefore$ percentage second harmonic distortion $\underline{= \ 0\%}$ Answer

The peak value of the fundamental component of the output $= 2 \times 35$

$= 70$ mA

and that of the third harmonic $2 \times 2 = 4$ mA

$\therefore$ percentage third harmonic distortion $= \frac{4}{70} \times 100\%$

$\underline{= \ 5.71\%}$ Answer

PUSH-PULL AMPLIFIERS WITH CLASS B BIAS

The operation of a push-pull amplifier under Class A conditions involves a large steady anode current which is the same whether the input signal is large or small, or if there is no input signal at all. This results in a waste of power and has led to the introduction of push-pull amplifiers working under Class B conditions.

When working with Class B bias (biased to cut-off) each valve contributes one-half of the output wave, the two halves being combined in the output transformer to give the original waveform.

The amount of a.c. power that may be obtained from the d.c. supplied to the stage is greater under Class B conditions than Class A. For this reason Class B bias is generally used where large amounts of audio power are required.

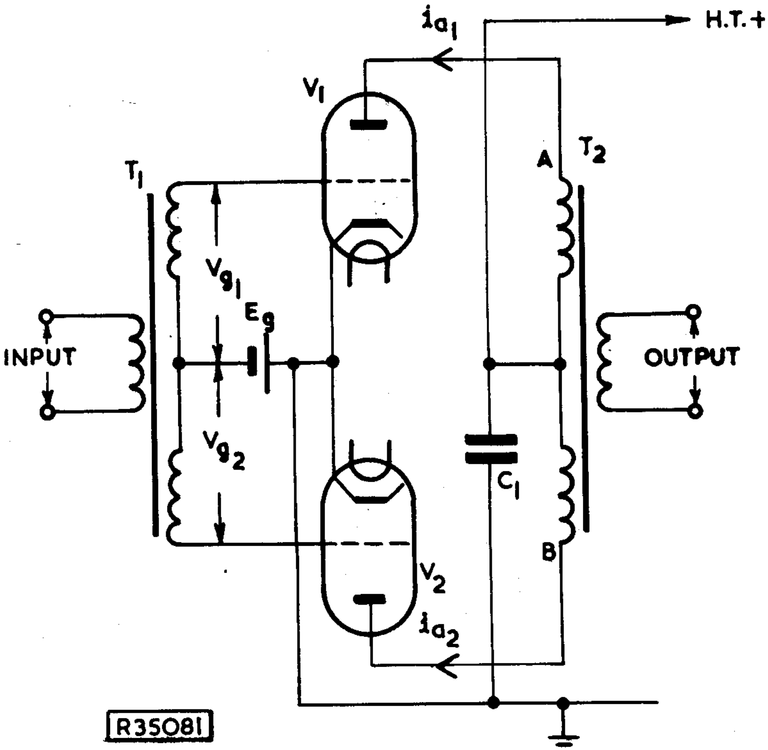
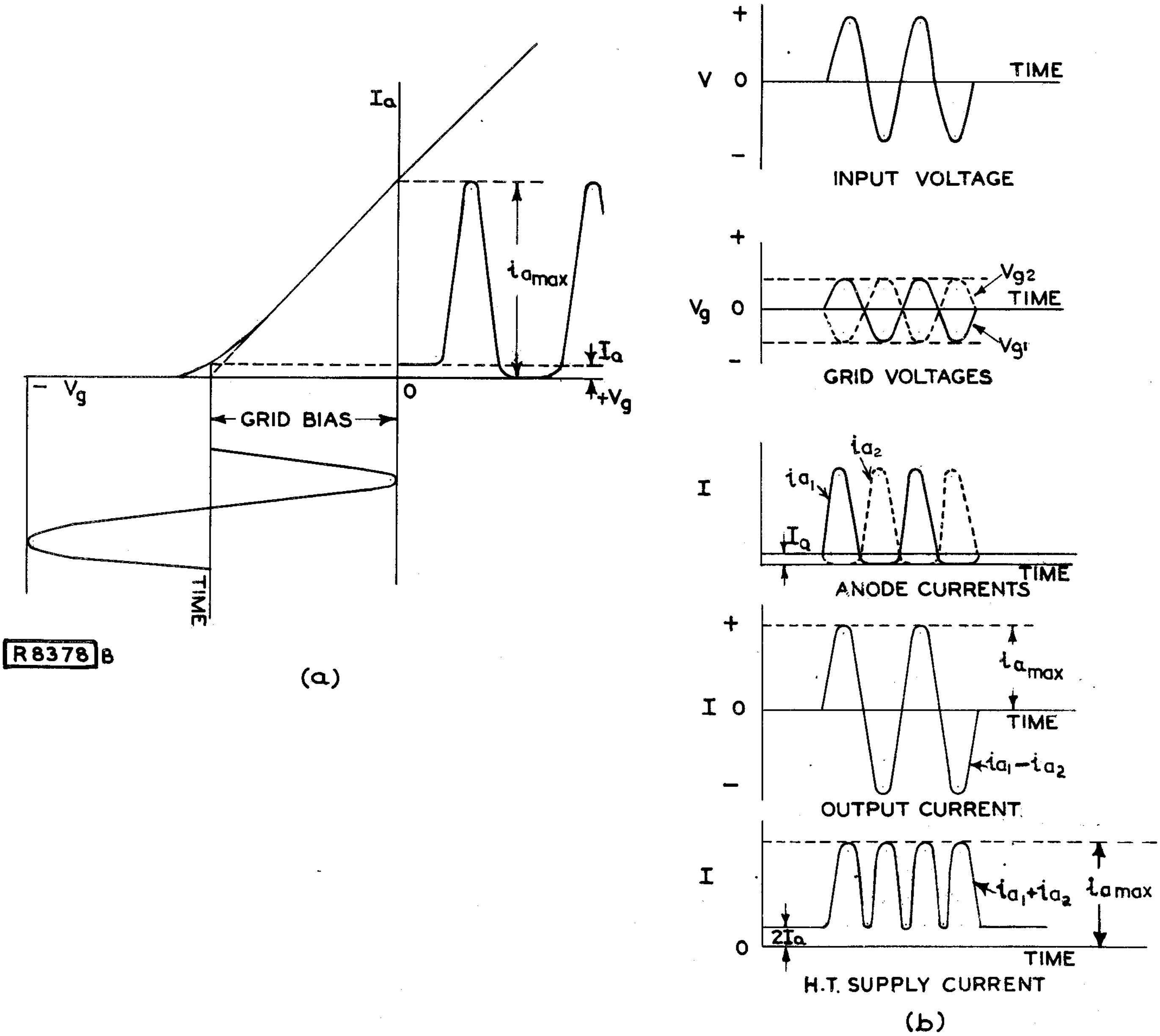


Fig. 12

A typical Class B push-pull circuit is shown in Fig. 12 where C_1 is the anode decoupling capacitor and the grid bias voltage is obtained from a fixed voltage supply.

The direct current taken from the anode power supply increases directly with the power output. For this reason, the method of obtaining the grid-bias voltage by means of a resistor and by-pass capacitor in the cathode lead cannot be used. The fixed value of grid-bias voltage must therefore be obtained from a separate rectifier unit or a battery. The main rectifier unit supplying the anodes must have very good regulation, to prevent the variations in the d.c. component of the anode current varying the supply voltage by more than a few volts.

The operating point on the dynamic characteristic of each valve is shown in Fig. 13(a) and the relationship between signal voltage and anode currents in Fig. 13(b). These should be compared with the corresponding curves for Class A operation shown in Fig. 3.



(a)

(b)

Fig. 13

It will be noticed that the grid bias voltage is slightly less than that required to bias the valve to cut-off. This has the effect of reducing the harmonic distortion introduced by the amplifier as will be shown later in the pamphlet.

In practice, Class B push-pull amplifiers are often operated with the output valves driven into the region of grid current in order to obtain a greater grid swing and consequent greater power output. This means that the push-pull stage must be supplied by a driver stage capable of delivering power and not just voltage. It is also necessary to keep the impedance of the output valve grid circuits as low as possible to prevent clipping of the positive half-cycles of the signal applied to each valve.

If the valves are to be driven into grid current the following factors must be considered.

(a) The anode voltage of each valve must not fall below a certain value or the flow of grid current will become excessive. This is illustrated by Fig. 14 which shows that if the anode voltage, E_a, becomes less than E_{min} a rapid increase in grid current will result.

(b) The grid dissipation must be kept to within the figure quoted by the valve manufacturer.

(c) The power required at the grids of the output valves for maximum positive grid voltage must not be greater than the maximum power output of the driver stage.

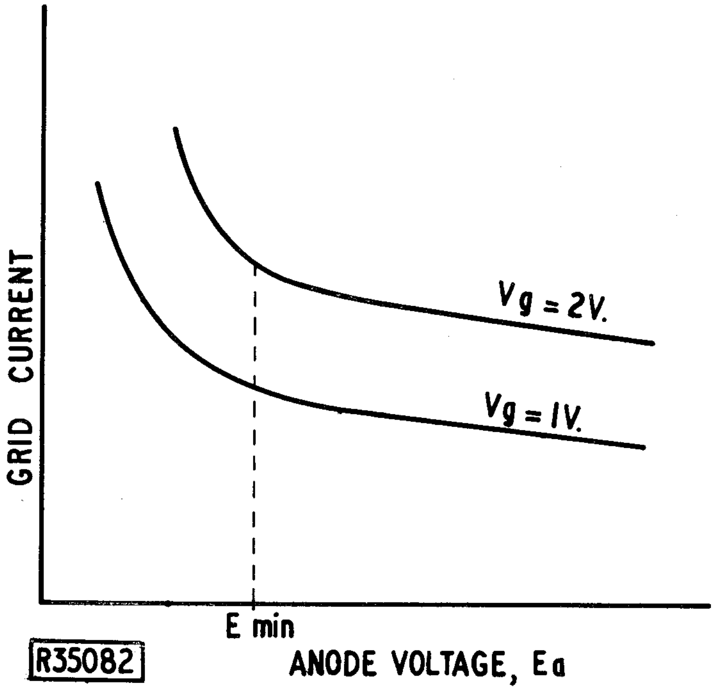


Fig. 14

DESIGN OF CLASS B AMPLIFIERS

The method used to draw composite characteristics for a Class B push-pull amplifier is the same as that employed for Class A amplifiers.

Consider the triode valve characteristics shown in Fig. 5 and assume that the grid bias voltage has been increased to -30 volts, the h.t. supply remaining constant at 250 volts. The valve is then biased to cut-off, i.e. Class B bias. The increase in grid bias moves the operating points, P_1 and P_2 of Fig. 6, vertically from the -20 volt characteristics to the -30 volt characteristic. Thus the operating points lie on the horizontal axis and coincide.

Fig. 15 shows composite characteristics drawn for the static point of the composite valve ($V_s = 0$) and for grid voltages 10 volts on either side ($V_s = \pm 10$)

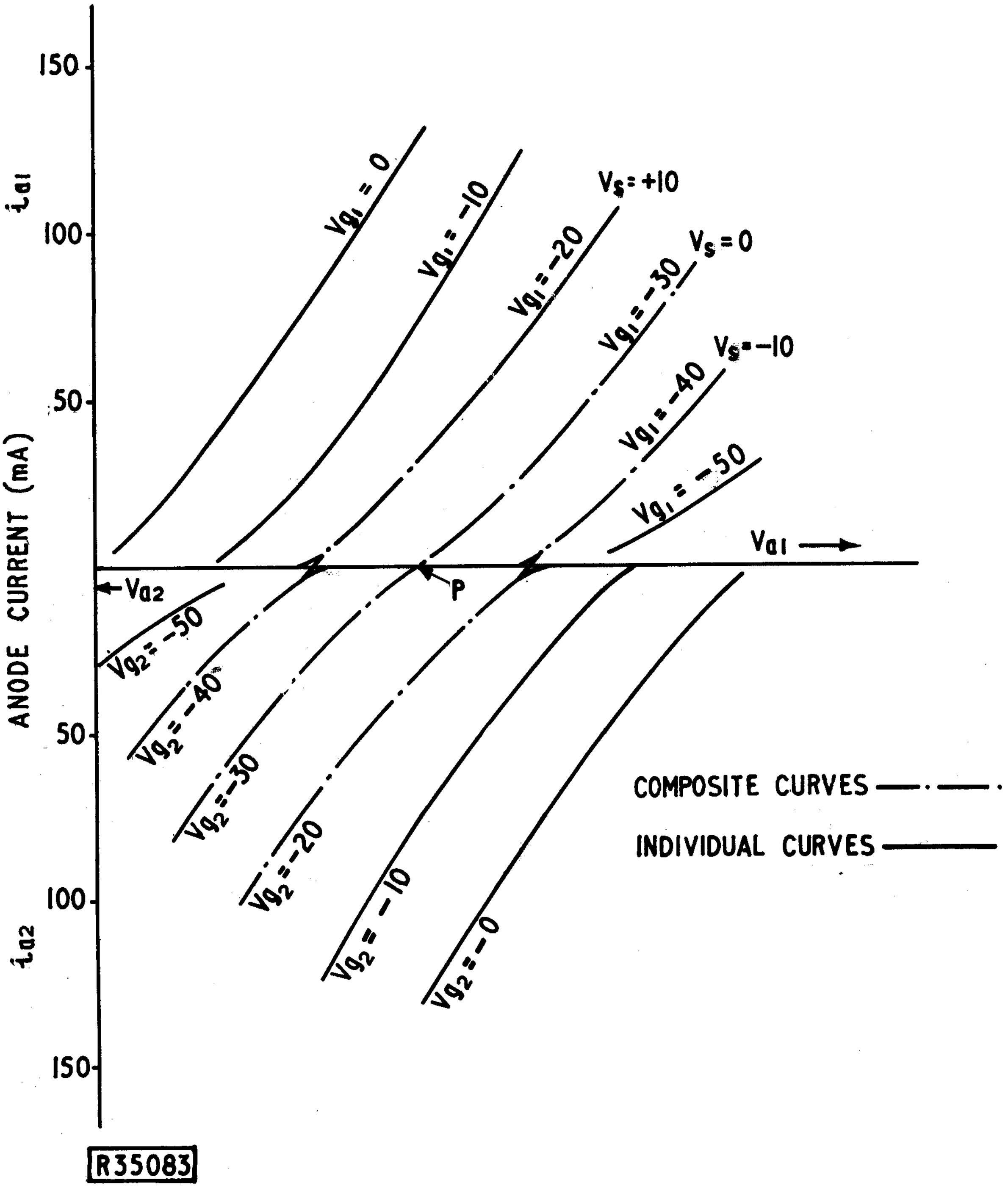


Fig. 15

It can be seen that when V_s is +10 volts the voltage applied to the grid of V_1 is $-30 + 10 = -20$ volts and that applied to the grid of V_2 is $-30 - 10 = -40$ volts. Similarly, on the negative half-cycle of the input signal the voltage applied to the grid of V_1 is -40 volts and that applied to the grid of V_2 is -20 volts.

It can also be seen that the composite characteristics follow the individual valve characteristics over most of their range, but owing to the considerable curvature of the individual characteristics in the region of low anode current, are appreciably curved. Over the range of values for which the composite characteristics follow the individual characteristics the slope is the same. Thus the anode slope resistance, r_a, of the composite valve is that of either valve and <u>not</u> $\frac{r_a}{2}$ as is the case for Class A operation.

It can be seen from Fig. 15 that the composite characteristics are kinked, if however, the negative bias is reduced slightly the composite curves become more linear. The value of grid bias that will give the straightened composite characteristics can be found by plotting the anode current/grid voltage characteristic for one valve at the normal h.t. voltage and projecting the straight portion of the curve to the horizontal axis. Fig. 16 has been plotted in this way. With a constant anode voltage of 250 volts, corresponding values of anode current and grid voltage have been taken from Fig. 15 and plotted. The straight portion of the curve has been extended to cut the horizontal axis from which it can be seen that the value of the projected grid voltage is -25 volts.

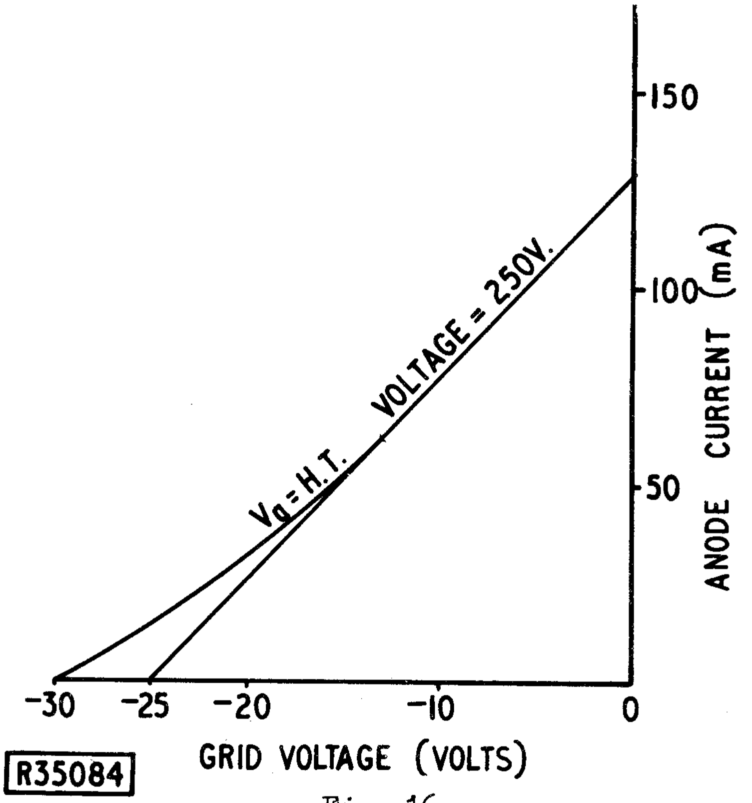


Fig. 16

Fig. 17 shows composite characteristics plotted for this new grid bias voltage and also for grid voltages 10 volts either side. The composite characteristics are more linear than those shown in Fig. 15 but not as linear as those for Class A bias as shown in Fig. 6.

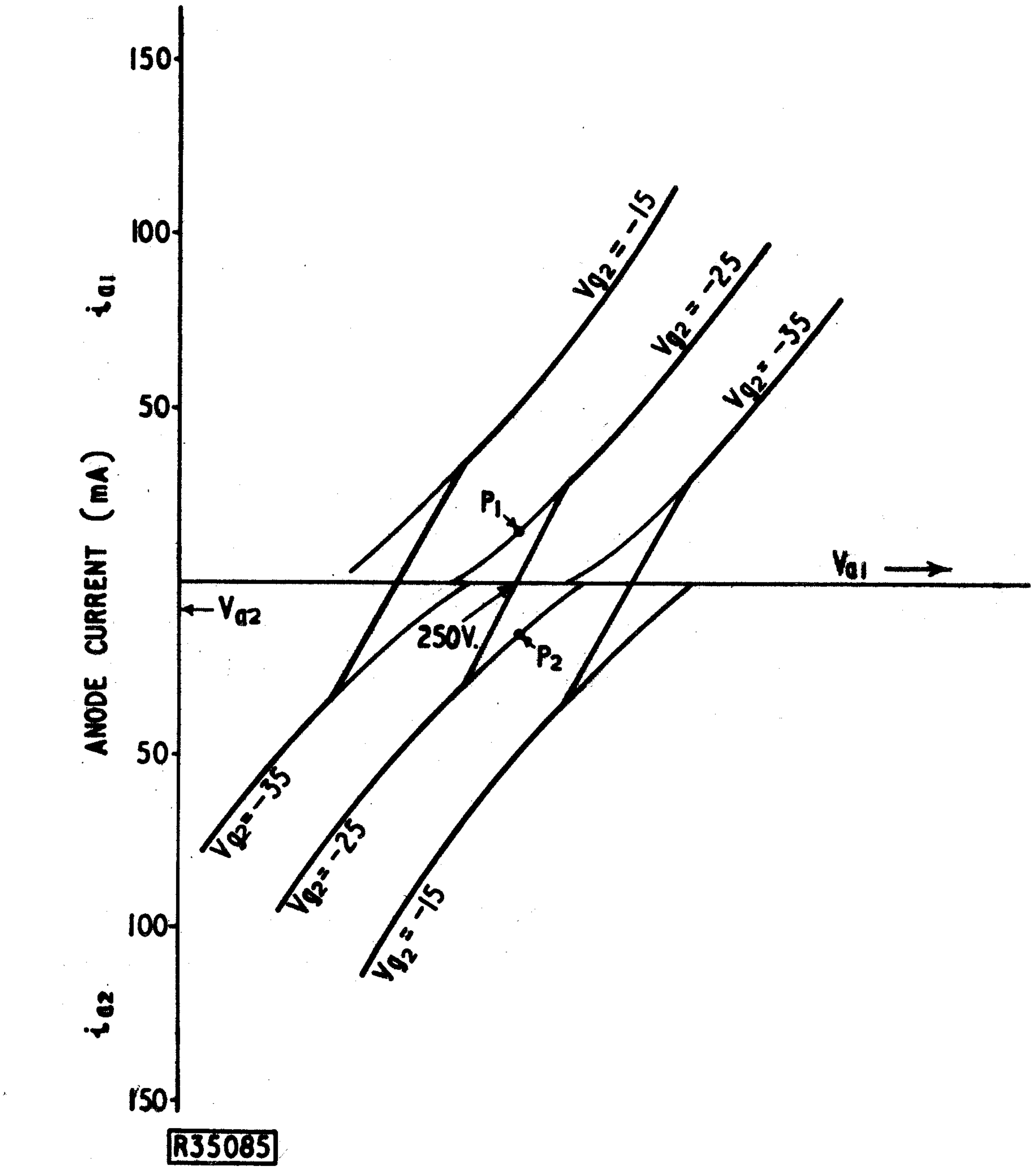


Fig. 17

Optimum load

It is shown in Appendix C that the Class B composite valve operates into the same load as the Class A composite valve, namely $\frac{R_L}{4}$, where R_L is the load between points A and B (Fig. 12).

For triodes, maximum power output is obtained when the composite load resistance, $\frac{R_L}{4}$, is equal to the anode slope resistance, r_a, of the composite valve,

$$\text{or} \quad \frac{R_L}{4} = r_a$$

$$R_L = 4\, r_a$$

But the anode slope resistance of the composite valve is equal to the anode slope resistance of either valve. Thus the load between points A and B should be equal to four times the anode slope resistance of either valve.

Pentodes and beam tetrodes in Class B push-pull

Pentodes and beam tetrodes may also be employed in Class B push-pull circuits, the composite characteristics etc. being obtained in similar manner to that employed for triode valves. The optimum load is chosen to reduce to a minimum the third harmonic content of the output because the second harmonic is reduced by the push-pull connexion.

EFFICIENCY OF CLASS B PUSH-PULL AMPLIFIERS

The anode current of each valve of a Class B push-pull pair is in the form of a series of pulses as shown in Fig. 13. The power supplied to each valve is equal to the product of the mean anode current and the h.t. supply voltage. It is shown in Appendix D that if the pulses of anode current are assumed to be half sine waves, the anode current may be represented by the expression:-

$$i_a = \frac{I_{max}}{\pi} + \frac{I_{max} \sin \theta}{2} - \frac{2\, I_{max} \cos 2\theta}{3\pi} - \dots\dots\dots\dots\dots$$

where I_{max} is the peak value of the pulses.

The mean value of the anode current over 1 cycle is $\frac{I_{max}}{\pi}$, thus the power supplied to either valve of the push-pull pair is $\frac{V_a I_{max}}{\pi}$ watts.

where V_a is the h.t. supply voltage.

Hence the total power supplied to the stage is $\frac{2\, V_a\, I_{max}}{\pi}$ watts. The component of the anode current at the fundamental frequency, for either valve, is $\frac{I_{max}}{2\sqrt{2}}$ r.m.s. and the r.m.s. value of the alternating anode voltage is $\frac{V_a}{\sqrt{2}}$. Thus the maximum power output of one valve is $\frac{I_{max}}{2\sqrt{2}} \times \frac{V_a}{\sqrt{2}}$.

The power output of both valves is $\frac{2\, I_{max}}{2\sqrt{2}} \times \frac{V_a}{\sqrt{2}}$

$$= \frac{V_a\, I_{max}}{2} \text{ watts}$$

The maximum efficiency of the stage is thus:-

$$\frac{V_a\, I_{max}}{2} \times \frac{\pi}{2\, V_a\, I_{max}} \times 100\% = 78.5\%$$

This value, however, may be approached only when the peak alternating anode voltage approximates to the quiescent value, and in practice the efficiency is usually limited to about 60%.

The efficiency when using pentode or tetrode valves is less than when using triode valves owing to the additional power dissipation at the screen grid.

Example

A Class B audio-frequency amplifier is to be designed using a pair of tetrode valves whose characteristics are shown in Fig. 18. The load connected to the secondary of the output transformer will be a 100 ohm resistor and the valves are to be driven up to peak grid voltages of +10 volts. An h.t. supply of 400 volts will be used.

Draw a suitable circuit of the output stage and estimate for maximum output and minimum distortion (a) the grid bias, (b) the output transformer turns ratio, (c) the maximum power output and (d) the current taken from the h.t. supply at maximum output.

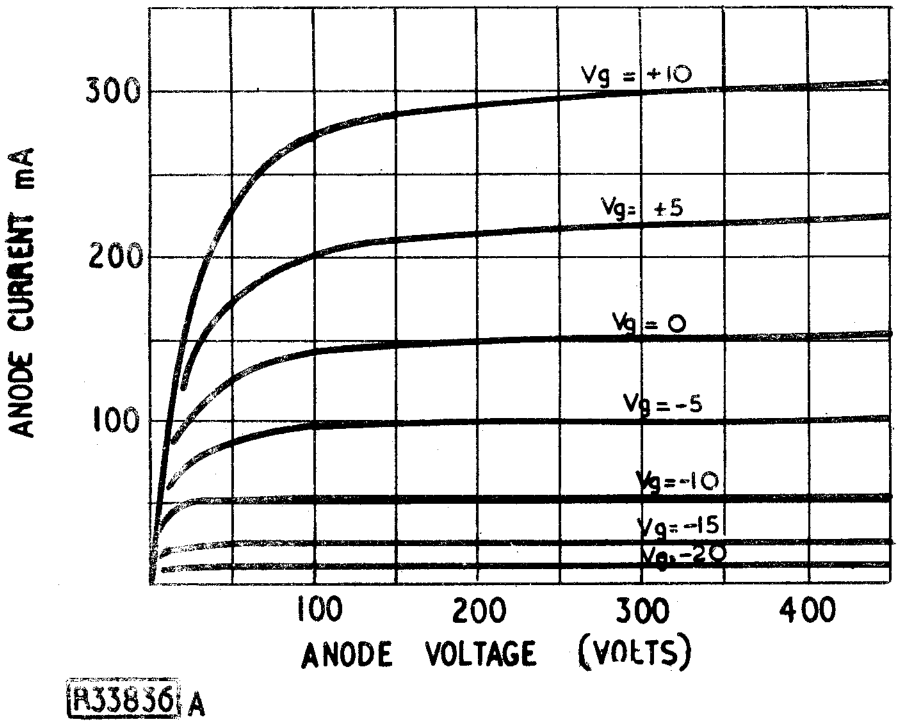


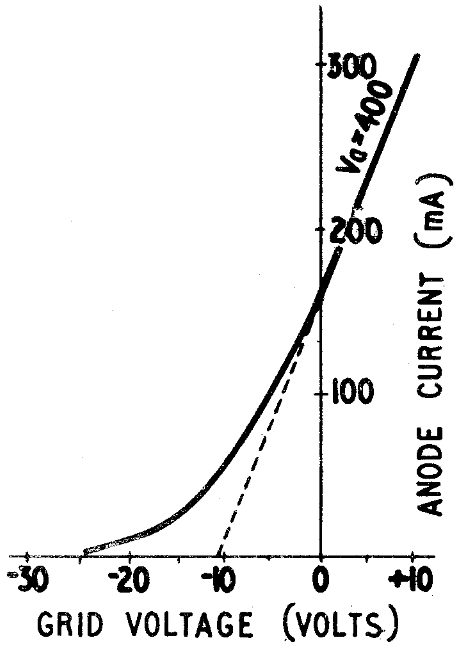
Fig. 18

The static characteristic of the valve for an anode voltage of 400 volts is drawn from the I_a/V_a characteristics shown in Fig. 18.

The value of the grid bias, found by projecting the straight portion of the curve to the horizontal axis, gives minimum distortion.

From Fig. 19

grid bias $\hat{=}$ -10 volts Answer (a)



R33837 A

Fig. 19

Two sets of static characteristic curves are plotted in Fig. 20, such that the horizontal axes coincide at the working anode voltage of 400 volts. Composite characteristic curves are drawn by taking the difference between the anode currents of the two valves at corresponding points on pairs of static characteristic curves for grid voltages equal amounts above and below the working bias of -10 volts.

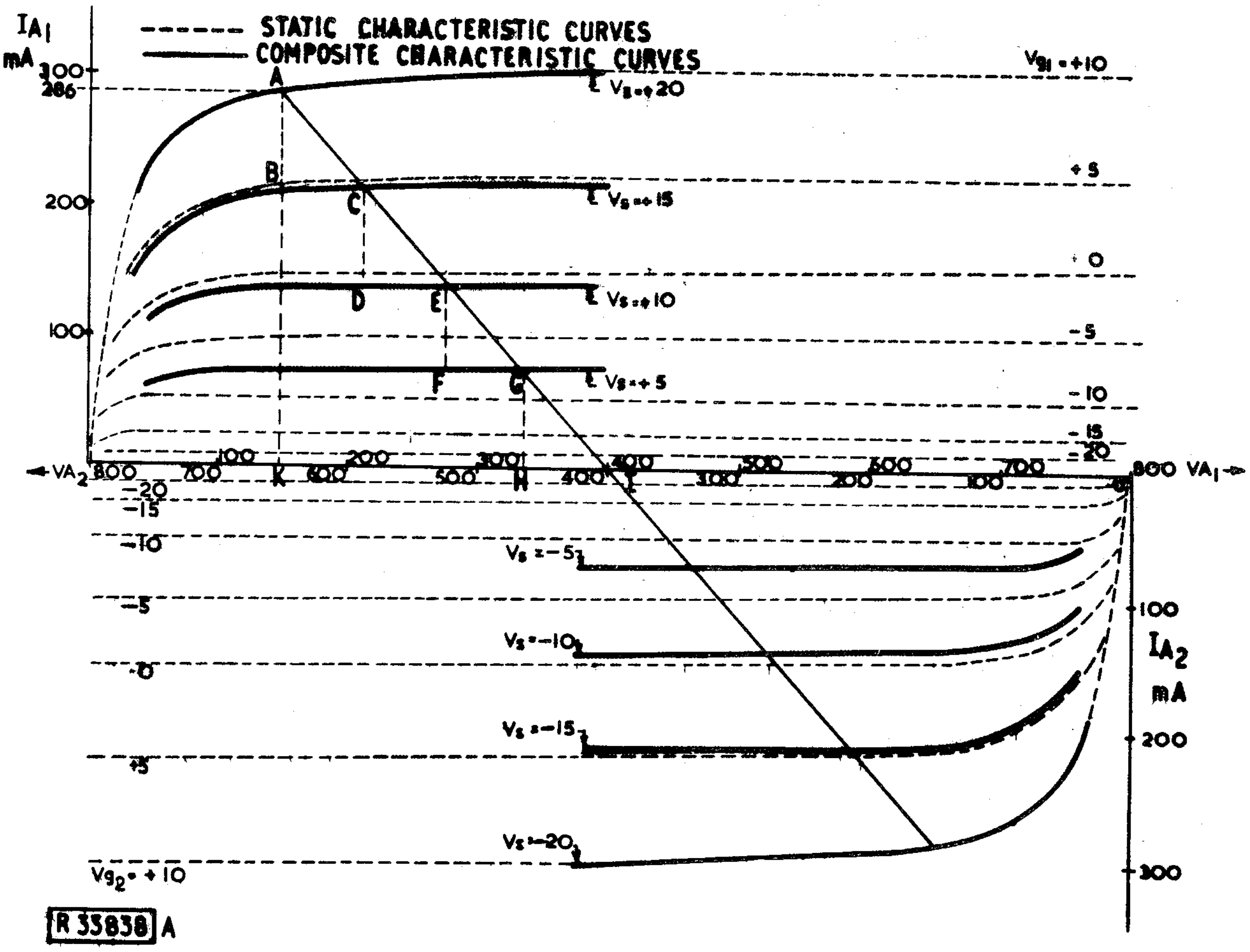

STATIC CHARACTERISTIC CURVES
COMPOSITE CHARACTERISTIC CURVES
IA1 mA
300
286
200
100
A
B
C
D
E
F
G
K
H
I
Vg1 = +10
+5
0
-5
-10
-15
-20
Vs = +20
Vs = +15
Vs = +10
Vs = +5
VA2 800 700 600 500 400 300 200 100
100 200 300 400 500 600 700 800 VA1
Vs = -5
Vs = -10
Vs = -15
Vs = -20
-20
-15
-10
-5
0
+5
Vg2 = +10
IA2 mA
100
200
300
R 33838 A

Fig. 20

A load line is chosen such that the intercepts AB, CD, EF and GH are approximately equal. This load line represents the load which gives maximum output with minimum distortion.

From Fig. 20 it can be seen that the slope of the load line is given by $-\frac{AK}{IK}$

$$\text{or } \frac{-0.286}{250}$$

$$= -1.144 \text{ mA/volt}$$

$$\text{Thus } \frac{-4}{R_L} = -1.144 \times 10^{-3}$$

where R is the load between points A and B (Fig. 21)

$$\text{hence } R_L = \frac{4000}{1.144}$$

$$= 3500 \text{ ohms}$$

and the turns ratio, n, of the output transformer is,

$$n = \sqrt{\frac{3500}{100}} \text{ (as the secondary load is 100 } \Omega\text{.)}$$

$$\underline{= 5.9 : 1}$$ Answer (b)

$$\text{Maximum power output} = \frac{0.286}{\sqrt{2}} \times \frac{250}{\sqrt{2}}$$

$$\underline{= 35.75 \text{ watts}}$$ Answer (c)

The current taken from the h.t. supply at maximum output is, from Fig. 20, 286 mA Answer (d)

Fig. 21 shows a suitable circuit for the output stage. The turns ratio of the input transformer, T_1, being such that the total peak secondary voltage is 40 volts with maximum input signal.

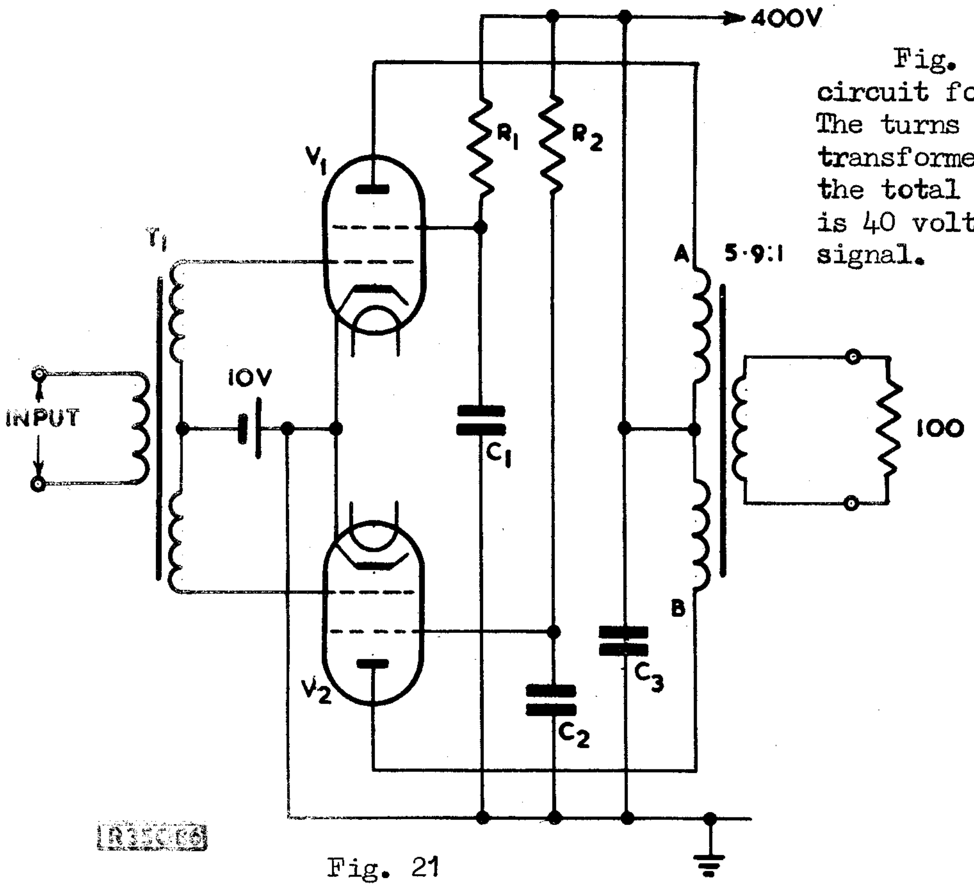


Fig. 21

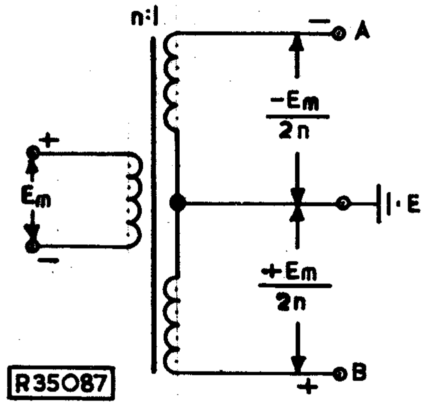
PHASE-SPLITTING CIRCUITS

An essential requirement for the operation of a push-pull amplifier is the splitting of the input signal into two separate voltages equal in magnitude but opposite in phase. Circuits that will do this are commonly known as phase-splitting circuits or sometimes phase inverters.

Phase-splitting circuits may be divided into two main types: those which are required to supply power to the push-pull stage and those which are not. A phase-splitter is required to deliver power and not just voltage whenever the output valves are driven into the region of grid current. This requires that the grid circuits of the output valves have a very low impedance so that the grid current flowing in the input circuit does not cause excessive harmonic distortion. Thus the phase-splitter for a Class B push-pull stage (involving grid current) is usually a step-down transformer with a centre-tapped secondary.

TRANSFORMER PHASE-SPLITTER

The simplest phase-splitting circuit is the centre-tapped transformer. Fig. 22 shows a transformer of turns ratio n : 1 having a centre-tapped



secondary. Consider the half-cycle of the input signal when the polarity is as shown. At this instant terminal A is negative with respect to earth and terminal B is in antiphase with the voltage at B. With an input voltage of E_m sin ωt the voltages across terminals AE and terminals BE have the same magnitude, $\frac{E_m \sin\omega t}{2n}$, and are in antiphase, provided the secondary winding has been accurately centre-tapped.

Fig. 22

The circuit of a push-pull amplifier employing transformer phase-splitting is shown in Fig. 23.

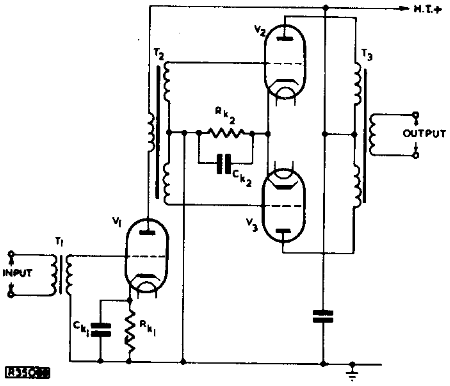


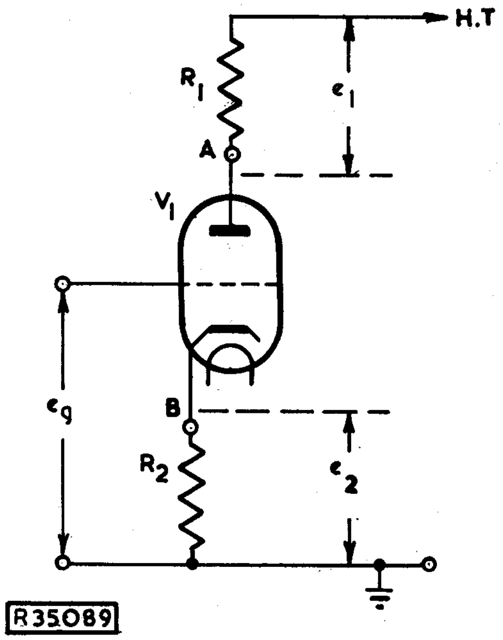
Fig. 23

If the push-pull output stage is working under Class B conditions, V_1 will be a power amplifier working under Class A conditions.

Transformer phase-splitting may also be used when the push-pull stage is working under Class A conditions, when the phase-splitter is operated as a transformer coupled voltage amplifier.

Transformer coupling, however, has the disadvantage that transformers are expensive and of limited frequency range, and thus other methods of phase splitting, not employing transformers, are generally used for Class A push-pull amplifiers.

PHASE-SPLITTER



A circuit which will supply two outputs 180° out of phase is shown in Fig. 24. An alternating voltage applied to the grid of V_1 will cause alternating voltages, e_1, and e_2, to be developed across resistors R_1 and R_2. If R_1 and R_2 are equal in value, voltages e_1 and e_2 will also be equal. Since R_1 is in the anode lead and R_2 is in the cathode lead, e_1 and e_2 are in antiphase when considered with respect to earth.

This circuit may be connected to a push-pull stage by connecting the earth line to the common cathodes and points A and B to the separate grids of the two output valves.

Fig. 24

Fig. 25 shows the 'Split Load Phase-Splitter' connected to a push-pull stage.

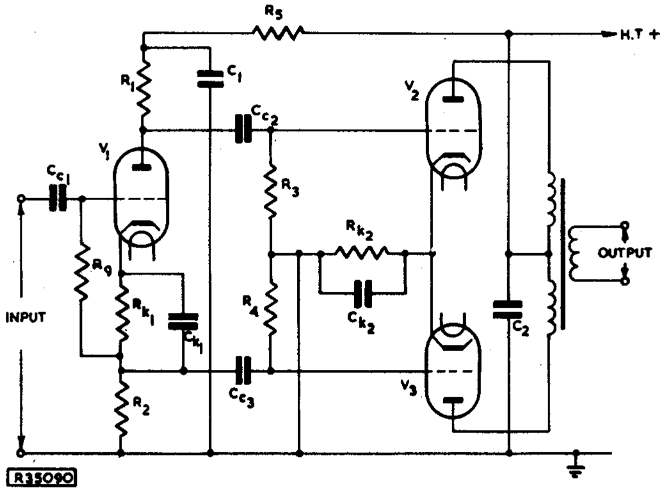


Fig. 25

The half of the phase-splitter feeding output valve V_3 is operated as a form of cathode follower and thus has a voltage gain of less than one. Since resistor R_1 is arranged to have the same value as R_2, the voltage gain of the other half of the phase-splitter is also less than one. Since this circuit is primarily intended to change a single input signal into two signals in antiphase, this is not necessarily regarded as a disadvantage.

FLOATING PARAPHASE PHASE-SPLITTER

The circuit of the 'Floating Paraphase Phase-Splitter' is shown in Fig. 26. A fraction, $\frac{R_5}{R_5 + R_6}$, of the output voltage of the first stage is applied to the grid of V_2. Thus the output voltage of the second stage is 180° out of phase with the output of the first stage. Consequently the voltage across R_5 will be reduced by a fraction of this out of phase voltage. This may be more easily seen by consideration of the equivalent circuits of the two stages shown in Figs. 27(a) and (b), which show the polarities at various points in the circuit for one half-cycle of the input signal.

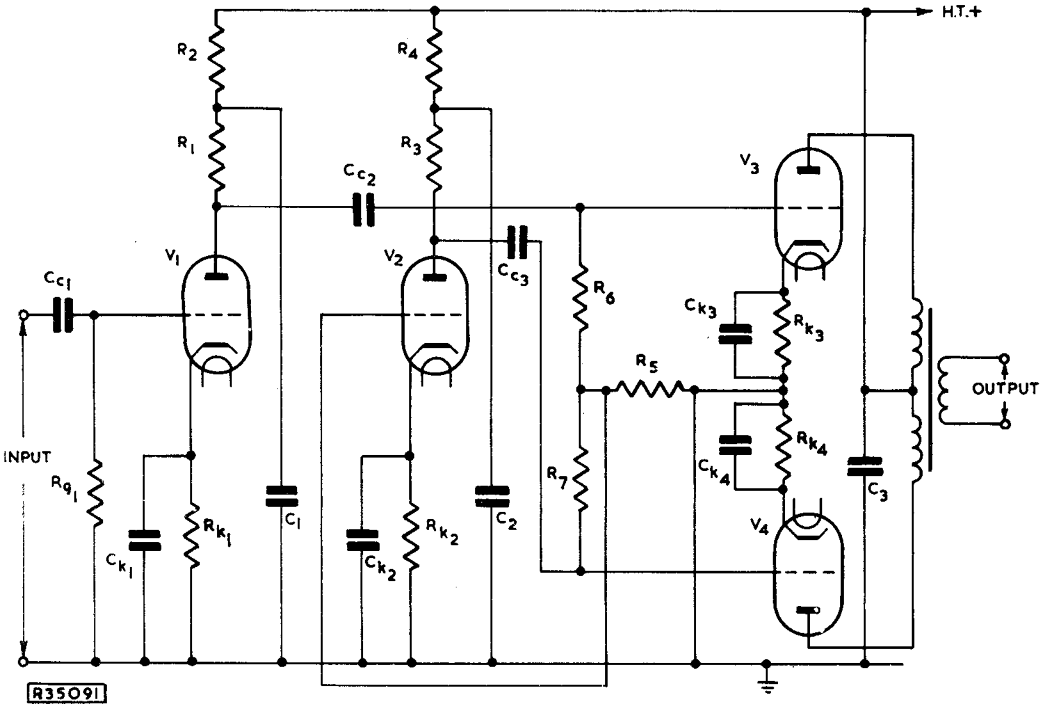


Fig. 26

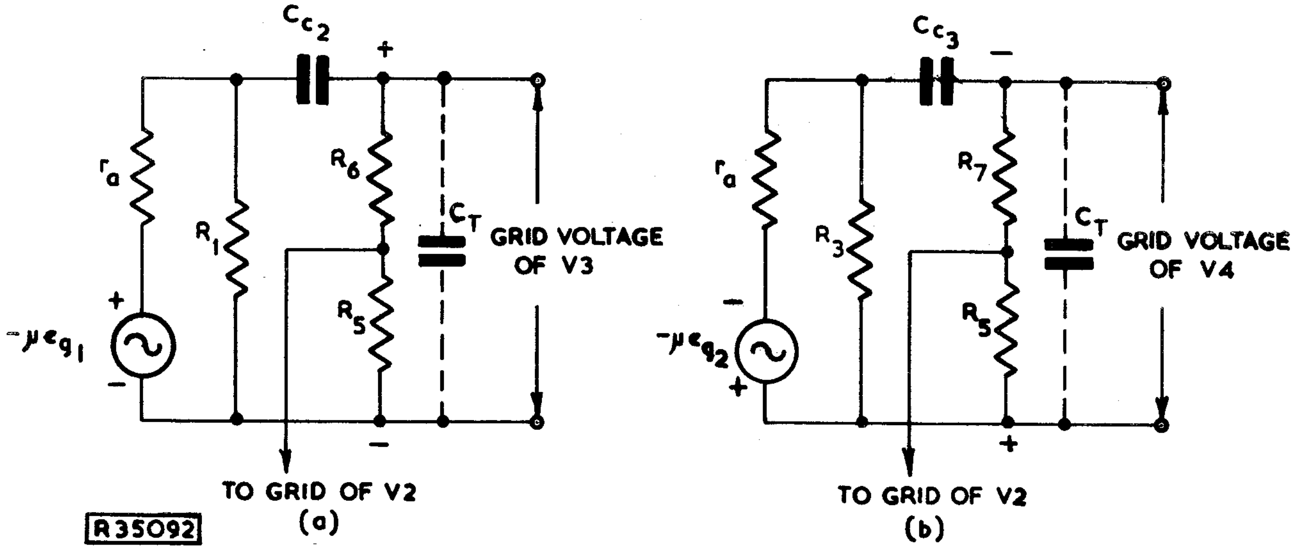


Fig. 27

If resistors R_6 and R_7 are approximately equal in value it is found that the voltages applied to the grids of V_3 and V_4 are 180° out of phase and their magnitudes are within 10% of each other.

CATHODE-COUPLED PHASE SPLITTER

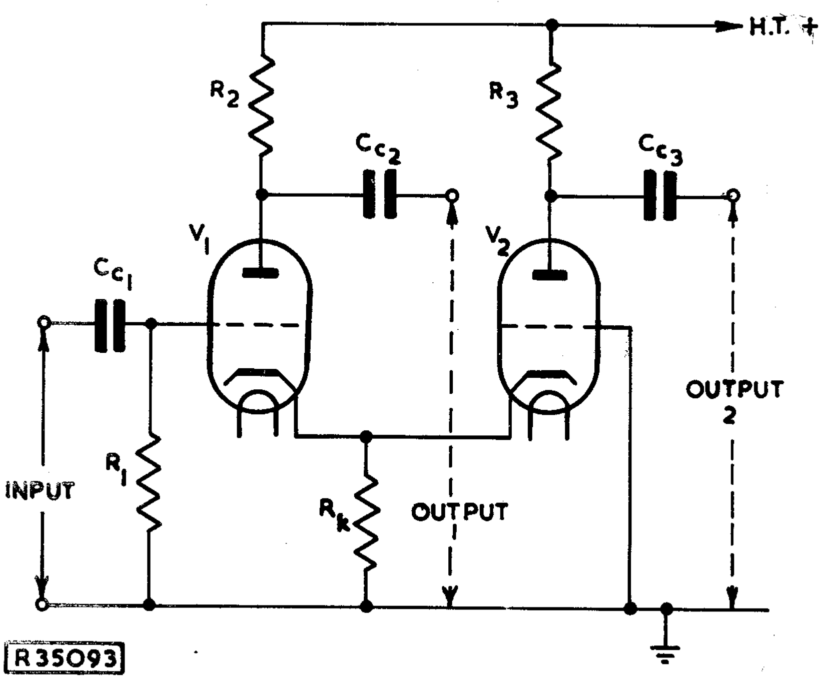


Fig. 28

Fig. 28 shows the circuit of a cathode-coupled phase splitter which consists of two valves coupled together by means of a common cathode resistor, R_K.

Consider the signal applied between the grid and cathode of V_1 to be increasing in a positive direction, thus increasing the current through the common cathode resistor, R_K. Since R_K is common to both valves this makes the voltage applied between the grid and cathode of V_2 increase in a negative direction. Thus the grid potentials of V_1 and V_2 are in antiphase. The two outputs of the phase-splitter are fed to the output stage via capacitors C_{C2} and C_{C3}.

The voltage developed across R_K also tends to oppose the signal applied between the grid and cathode of V_1. This is a form of negative-feedback and has the effect of reducing any distortion introduced by the circuit. Several other effects also occur but they will not be discussed as they are outside the scope of this pamphlet.

This circuit enables a high degree of push-pull balance to be obtained, provided resistors R_2 and R_3 are matched within 5%. A disadvantage of this circuit is that the effective voltage gain is only about half that obtained from one section when used as a normal voltage amplifier.

PRACTICAL PUSH-PULL AMPLIFIERS

Fig. 29 shows a simplified circuit of the amplifier section of the harmonic generator used in a typical carrier frequency generating equipment.

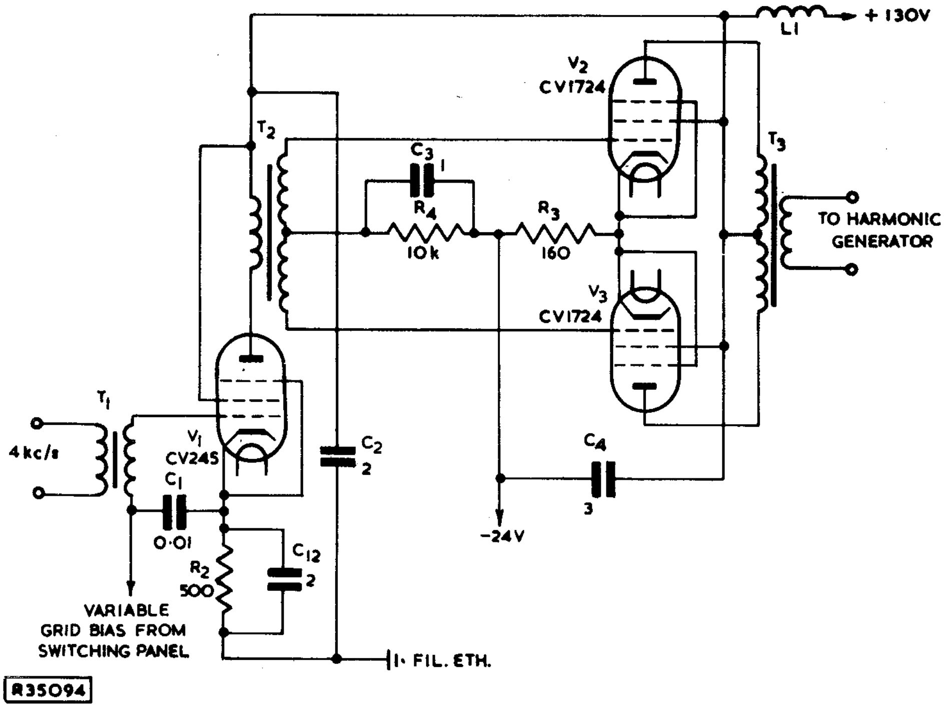


Fig. 29

The 4 kc/s input signal is applied to the grid of V_1, amplified, and transformer coupled to the push-pull stage. The secondary winding of transformer T_2 is centre-tapped and thus the grid voltages of V_2 and V_3 are in antiphase. The outputs of V_2 and V_3 are combined in the output transformer and applied to the harmonic generator proper.

Fig. 30 shows a push-pull amplifier providing a maximum output power of 15 watts with a reasonably flat gain/frequency characteristic from 10 - 20000 c/s.

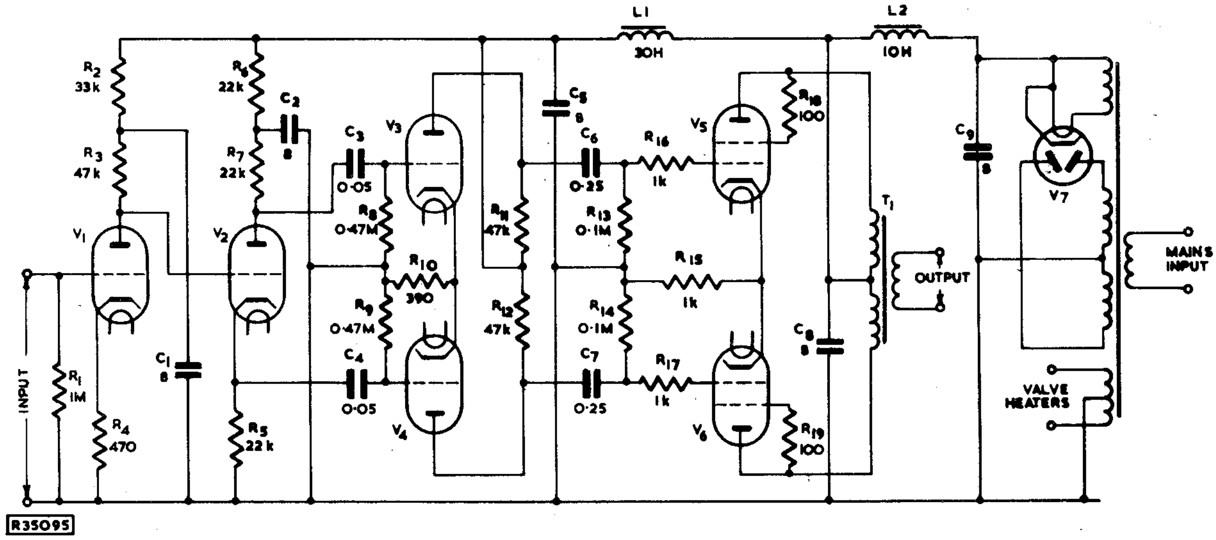


Fig. 30

Direct coupling is employed between the first stage and the phase-splitting stage. This eliminates one R.C. coupling and thus improves the gain/frequency characteristic of the amplifier at the lower frequencies. The phase-splitting stage is of the type shown in Fig. 24, and thus the voltages applied to the first push-pull stage are in anti-phase and of approximately the same magnitude.

The outputs of each valve of this stage are R.C. coupled to the final push-pull stage which is operated under Class A conditions.

Grid bias for the first push-pull stage is obtained from R_{10} and applied via R_8 and R_9. The grid bias for the final push-pull stage is obtained from R_{15} and applied to the grids of V_5 and V_6 via R_{13} and R_{14}.

The circuit of an amplifier having a push-pull output stage employing pentode valves working under distributed load conditions is shown in Fig. 31.

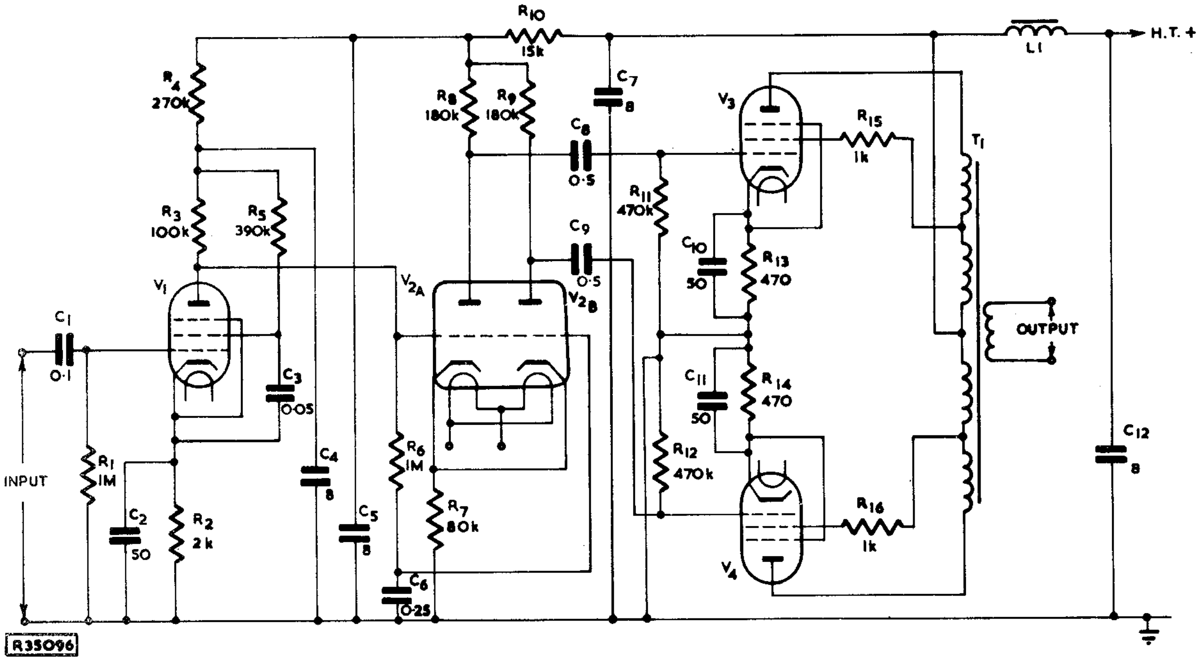


Fig. 31

The amplifier has a gain/frequency characteristic of ± 1 db of its nominal gain over the frequency range 10 c/s to 70 kc/s and ± 3 db over the frequency range 2 c/s to 100 kc/s.

The first stage is d.c. coupled to the phase-splitter in order to reduce the phase shift at low frequencies to a minimum. The phase-splitting stage is of the cathode-coupled type and fulfills the combined function of phase-splitter and amplifier. The output stage grid resistors, R_{11} and R_{12} must be accurately matched because they are effectively in parallel with the anode load resistors of the phase-splitter.

The balance of the phase-splitter is maintained at high frequencies by careful layout of components and wiring, and the balance at low frequencies by the inclusion of C_6 in the circuit.

The outputs of the phase-splitter are R.C. coupled to the push-pull output stage. The output stage is operated with the screen grids fed from tapping points on the primary winding of the output transformer. Approximately 40% of the primary winding is in circuit between the screen grids of V_3 and V_4. Resistors R_{15} and R_{16} are fitted as it is found that improved linearity results. The output valves have separate cathode resistances in order to reduce the out of balance d.c. component in the primary of the output transformer.

END

APPENDIX 'A'

ELIMINATION OF THE D.C. AND EVEN ORDER HARMONIC COMPONENTS OF THE OUTPUT OF A PUSH-PULL AMPLIFIER

As a first approximation the dynamic characteristic of a valve may be considered to be linear over most of the normal operating range, for more precise calculations the anode current must be expressed as a power series, i.e.

$$i_a = a + bVg + cVg^2 + dVg^3 + eVg^4 + fVg^5 + \ldots\ldots\ldots\ldots$$

where a, b, c etc. are constants and Vg is the total voltage applied to the grid (signal voltage plus bias voltage). It should be noted that sufficient accuracy is obtained for most purposes if the first four terms only are considered,

$$\text{or } i_a = a + bVg + cVg^2 + dVg^3$$

Now at any instant the signal voltage applied to the grid of one valve is in anti-phase with the signal voltage applied to the grid of the other,

$$\text{i.e. } eg_1 = -eg_2$$

Thus the anode current of one valve, V_1, is given by:-

$$i_{a1} = a + b(eg_1 - Eg) + c(eg_1 - Eg)^2 + d(eg_1 - Eg)^3$$

and the anode current of V_2 by:-

$$i_{a2} = a + b(-eg_2 - Eg) + c(-eg_2 - Eg)^2 + d(-eg_2 - Eg)^3$$

where Eg is the bias voltage.

These two currents flow in opposite directions in the primary winding of the output transformer, thus the effective primary current is $i_{a1} - i_{a2}$.

Consider a push-pull stage, the output valves of which have a bias voltage of -20 volts applied to the grids. Let the input signal be given by 10 sin ωt. Then,

$$i_{a1} = a + b(10 \sin \omega t - 20) + c(10 \sin \omega t - 20)^2 + d(10 \sin \omega t - 20)^3$$

$$= a + b(10 \sin \omega t - 20) + c(100 \sin^2 \omega t - 400 \sin \omega t + 400) +$$

$$d(10^3 \sin^3 \omega t - 6 \times 10^3 \sin^2 \omega t + 12 \times 10^3 \sin \omega t - 8 \times 10^3)$$

$$\therefore\ i_{a1} = a - 20b + 400c - 8 \times 10^3\ d + (10b - 400c + 12 \times 10^3 d) \sin \omega t +$$

$$(100c - 6 \times 10^3 d) \sin^2 \omega t + 10^3 d \sin^3 \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

$$\text{and } i_{a2} = a + b(-10 \sin \omega t - 20) + c(-10 \sin \omega t - 20)^2 + d(-10 \sin \omega t - 20)^3$$

$$= a + b(-10 \sin \omega t - 20) + c(100 \sin^2 \omega t + 400 \sin \omega t + 400) +$$

$$d(-10^3 \sin^3 \omega t - 6 \times 10^3 \sin^2 \omega t - 12 \times 10^3 \sin \omega t - 8 \times 10^3)$$

$$= a - 20b + 400c - 8 \times 10^3 d + (-10b + 400c - 12 \times 10^3 d) \sin \omega t +$$

$$(100c - 6 \times 10^3 d) \sin^2 \omega t - 10^3 d \sin^3 \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

$$\text{Then } i_{a1} - i_{a2} = \text{equation (1)} - \text{equation (2)}$$

$$= 2(10b - 400c + 12 \times 10^3 d) \sin \omega t + 2 \times 10^3 d \sin^3 \omega t$$

Thus the d.c. and even harmonic components cancel in the output transformer. This eliminates the problem of d.c. saturation of the transformer core, and reduces the total harmonic distortion. This permits output powers greater than double the output power of one valve for the same percentage distortion or a reduced total harmonic distortion for the same output per valve.

APPENDIX 'B'

EQUIVALENT CIRCUITS FOR CLASS A PUSH-PULL AMPLIFIERS

An equivalent circuit can be drawn for a push-pull amplifier working under Class A conditions provided the following assumptions can be made:-

(a) the valves are identical

(b) the valve characteristics are linear over the working range.

Since push-pull valves are usually driven over the full range of their characteristics, assumption (b) is not very accurate. Hence the more accurate graphical method, described in the pamphlet, is generally used when making calculations on push-pull amplifiers.

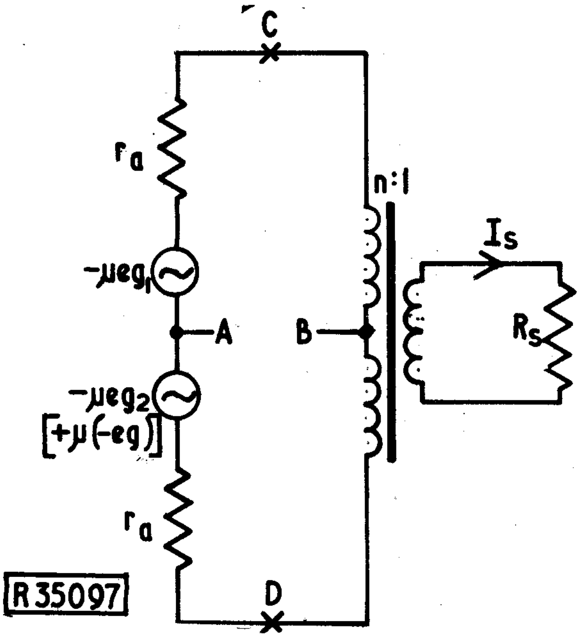


Fig. 32

The equivalent circuit of a Class A push-pull amplifier employing triode valves is shown in Fig. 32. It is assumed that the triode valves generate the fundamental frequencies only and that the output transformer is ideal. Points A and B are at the same potential as far as the fundamental frequency is concerned thus the connection between them may be omitted. The turns ratio, n:1, refers to the whole of the primary winding.

Let Fig. 32 be cut at points C and D and Thevenin's Theorem applied. Then the impedance seen looking into terminals C and D is $2r_a$ and the open circuit voltage is $-2\mu eg$. Thus Fig. 32 may be redrawn as shown in Fig. 33 and Fig. 34 shows the circuit referred to the secondary.

From Fig. 34,

$$I_s = \frac{-2\mu eg}{n\left(R_s + \frac{2r_a}{n^2}\right)} \quad \ldots\ldots\ldots\ldots(1)$$

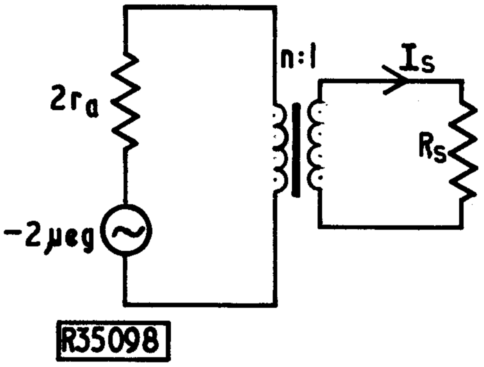


Fig. 33

If, now, Fig. 32 is redrawn with the two equivalent generators in parallel and connected to one half of the primary winding, Fig. 35 will result.

If Thevenin's Theorem is applied to points C and D of Fig. 35, then the impedance seen looking into terminals C and D is $\frac{r_a}{2}$ and the open circuit voltage is $-\mu eg$.

Thus Fig. 36 is equivalent to Fig. 35. Fig. 36 may be further simplified by referring the complete circuit to the secondary. This is shown in Fig. 37.

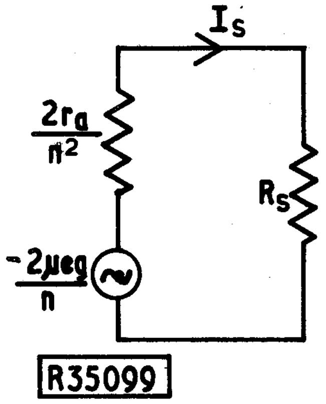


Fig. 34

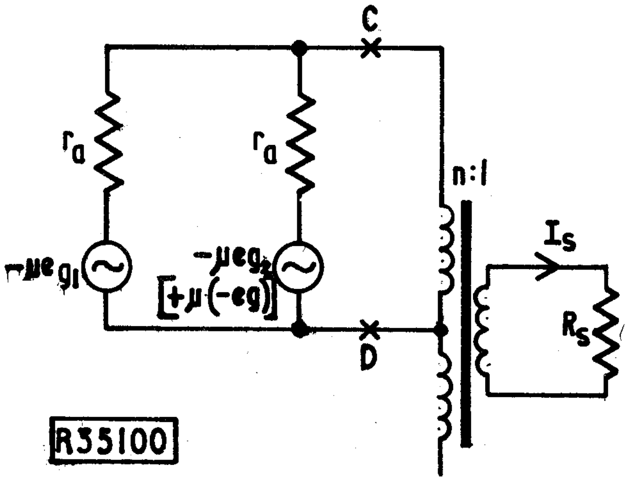


Fig. 35

From Fig. 37,

$$I_s = \frac{-2\mu e g}{n\left(R_s + \frac{2r_a}{n^2}\right)} \quad \ldots\ldots\ldots\ldots(2)$$

If equations (1) and (2) are compared it will seem that the two equations are identical. Thus, as far as the load is concerned, Figs. 32 and 35 are identical.

Fig. 36 is the equivalent circuit of the composite valve used in the graphical treatment of Class A push-pull amplifiers. It can be seen that the anode slope resistance of the composite valve is $\frac{r_a}{2}$, where r_a is the anode slope resistance of one valve. The amplification factor of the composite valve is the same as that of a single valve, i.e. μ. The load on the composite valve is $\left(\frac{n}{2}\right)^2 R_s = \frac{n^2 R_s}{4}$. The anode to anode load n^2R_s is known as R_L and thus the slope of the composite load line is $-\frac{4}{R_L}$.

For maximum power transfer between the composite valve and its load, $\frac{r_a}{2} = \frac{r_L}{4}$

$$\text{or } R_L = 2r_a$$

or the anode to anode load of a Class A push-pull amplifier should be twice the anode slope resistance of one valve.

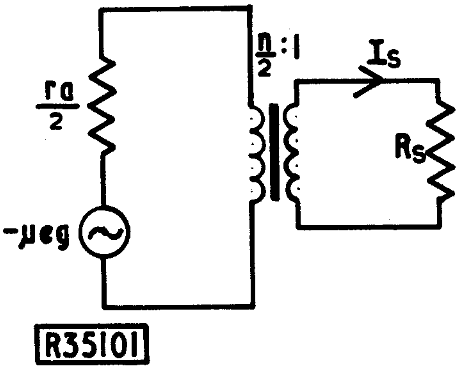


Fig. 36

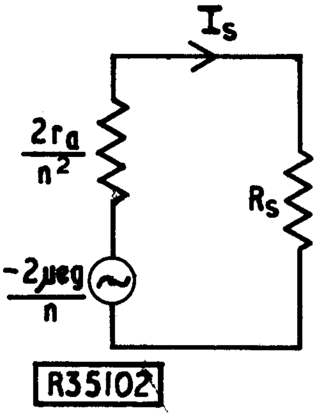


Fig. 37

APPENDIX 'C'

EQUIVALENT CIRCUITS FOR CLASS B PUSH-PULL AMPLIFIERS

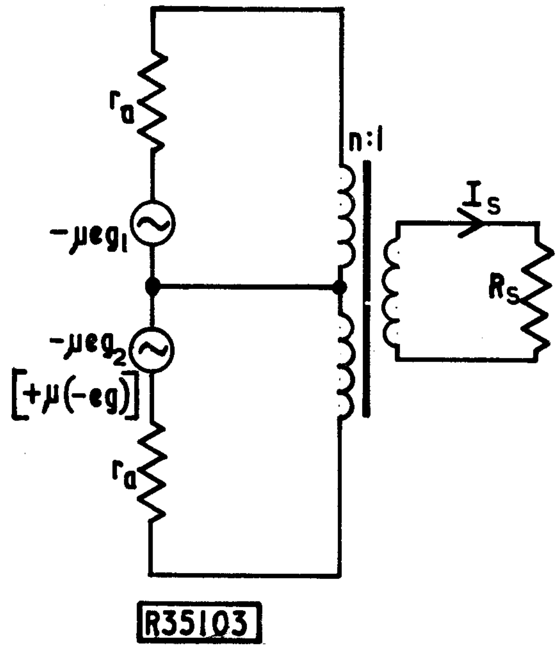


Fig. 38

The equivalent circuit of a Class B push-pull amplifier is shown in Fig. 38. Since each valve is biased to cut-off, the equivalent generators are each operative for alternate half-cycles only. It may therefore be assumed that each generator is open circuit during its 'cut-off' half-cycles. Thus Fig. 38 may be redrawn as shown in Fig. 39, where one generator supplies the positive half-cycles of the output and the other generator supplies the negative half-cycles.

Fig. 40 shows how these generators may be replaced by a single generator supplying both half-cycles. Fig. 40 shows the equivalent circuit of the composite valve used in the graphical analysis of Class B push-pull amplifiers. It can be seen that it has the same anode slope resistance, r_a, and amplification factor, μ, as a single valve. The composite valve operates into a load of $(\frac{n}{2})^2 R_s$ or $\frac{n^2 R_s}{4}$. The anode to anode load, $R_L = n^2 R_s$ and thus the slope of the composite load line is $-\frac{4}{R_L}$ as for a Class A amplifier.

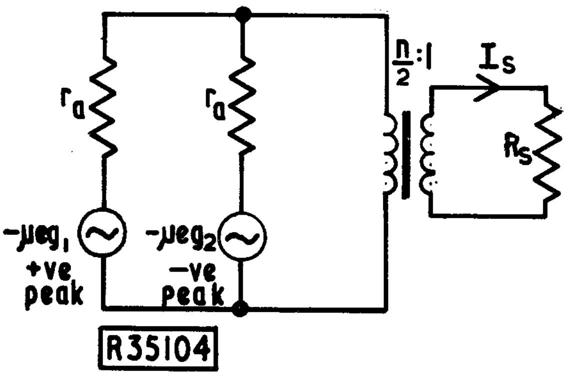


Fig. 39

For maximum power transfer between the composite valve and its load, $r_a = \frac{R_L}{4}$

$$\text{or } R_L = 4r_a$$

or the anode to anode load of a Class B push-pull amplifier should be four times the anode slope resistance of one valve. It should be noted that this is twice the load required for the same two valves operating under Class A conditions.

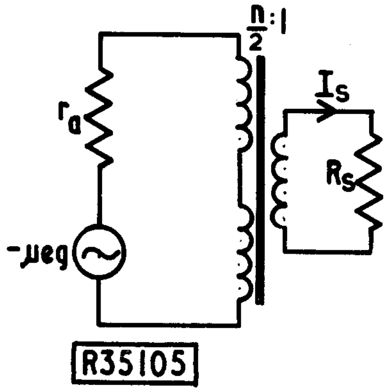


Fig. 40

APPENDIX 'D'

DERIVATION OF AN EXPRESSION FOR THE ANODE CURRENT OF A CLASS B AMPLIFIER

Fourier's Theorem states that any periodic function may be represented by the sum of a constant term plus all possible sine and cosine curves and may be expressed in the form,

$$y = f(\theta) = A_0 + A_1 \cos\theta + A_2 \cos 2\theta + A_3 \cos 3\theta + \ldots\ldots A_n \cos n\theta$$
$$+ B_1 \sin\theta + B_2 \sin 2\theta + B_3 \sin 3\theta + \ldots\ldots B_n \sin n\theta$$

This is the Fourier Series.

It can be shown that,

$$A_o = \frac{1}{2\pi}\int_o^{2\pi} y \, d\theta$$

$$A_n = \frac{1}{\pi}\int_o^{2\pi} y \cos n\theta \, d\theta$$

$$B_n = \frac{1}{\pi}\int_o^{2\pi} y \sin n\theta \, d\theta$$

The values of A_o, A_n and B_n are found using these integrals and substituted in the Fourier Series, to give the particular series for $y = f(\theta)$.

Fig. 41 shows the waveform of the anode current of a Class B amplifier. It is assumed that the pulses of anode current are half sine waves. Then the equation of the anode current is $f(\theta) = I_m \sin\theta$ in the range $0 \leqslant \theta \leqslant \pi$ and $f(\theta) = 0$ in the range $\pi \leqslant \theta \leqslant 2\pi$

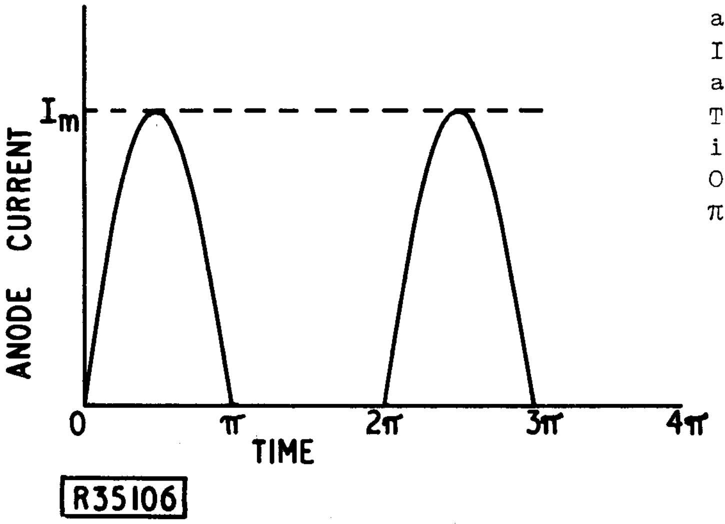


Fig. 41

$$A_o = \frac{1}{2\pi}\int_o^{2\pi} f(\theta)\ d\theta$$

$$= \frac{1}{2\pi}\left[\int_o^{\pi} I_m \sin\theta\ d\theta + \int_{\pi}^{2\pi} 0\ d\theta\right]$$

$$= \frac{1}{2\pi}\Big[- I_m \cos\theta \Big]_o^{\pi}$$

$$= \frac{1}{2\pi}\Big[(- I_m \cos\pi) - (- I_m) \Big]$$

$$= \frac{1}{2\pi}\Big[I_m + I_m \Big]$$

$$\therefore \quad A_o = \frac{I_m}{\pi}$$

$$A_n = \frac{1}{\pi}\int_o^{2\pi} f(\theta)\ \cos n\theta\ d\theta$$

$$= \frac{1}{\pi}\left[\int_o^{\pi} I_m \sin\theta \cos n\theta\ d\theta + \int_{\pi}^{2\pi} 0\ d\theta\right]$$

A trigonometrical identity is,

$$2 \sin\theta \cos n\theta = \frac{1}{2} \sin (1 + n)\theta + \sin (1 - n)\theta$$

hence $$A_n = \frac{I_m}{2\pi}\left[\int_o^{\pi} \{\sin (1 + n)\theta + \sin (1 - n)\theta\} d\theta\right]$$

$$= \frac{I_m}{2\pi}\left[- \frac{\cos (1 + n)\theta}{1 + n} - \frac{\cos (1 - n)\theta}{1 - n}\right]_o^{\pi}$$

$$= \frac{I_m}{2\pi}\left[\left(- \frac{\cos (1 + n)\pi}{1 + n} - \frac{\cos (1 - n)\pi}{1 - n}\right) - \left(- \frac{1}{1 + n} - \frac{1}{1 - n}\right)\right]$$

If n is odd, 1 ± n are both even and,

$$A_n = \frac{I_m}{2\pi}\left[-\frac{1}{1+n} - \frac{1}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

$\therefore A_n = 0$ when n is odd

If n is even, 1 ± n are both odd and,

$$A_n = \frac{I_m}{2\pi}\left[\frac{1}{1+n} + \frac{1}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

$$= \frac{I_m}{2\pi}\left[\frac{2}{1+n} + \frac{2}{1-n}\right]$$

$$= \frac{I_m}{\pi}\left[\frac{2}{1-n^2}\right]$$

$$\therefore A_n = \frac{2\,I_m}{\pi}\left[\frac{1}{1-n^2}\right] \text{ when n is even}$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\,\sin n\theta\,d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} I_m \sin\theta\,\sin n\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

Another trigonometrical identity is,

$$2\sin\theta\,\sin n\theta = \cos(1-n)\theta - \cos(1+n)\theta$$

hence
$$B_n = \frac{I_m}{2\pi}\left[\int_0^{\pi}\{\cos(1-n)\theta - \cos(1+n)\theta\}\,d\theta\right]$$

$$= \frac{I_m}{2\pi}\left[\frac{\sin(1-n)\theta}{1-n} - \frac{\sin(1+n)\theta}{1+n}\right]_0^{\pi}$$

$$= \frac{I_m}{2\pi}\left[\frac{\sin(1-n)\pi}{1-n} - \frac{\sin(1+n)\pi}{1+n}\right]$$

If n is odd, $1 \pm n$ is even and $B_n = 0$ except when $n = 1$.

When $n = 1$,

$$B_n = \frac{I_m}{\pi}\int_0^{\pi} \sin^2\theta \, d\theta$$

$$= \frac{I_m}{2\pi}\left[\int_0^{\pi}(1 - \cos 2\theta)\, d\theta\right]$$

$$= \frac{I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$\therefore B_n = \frac{I_m}{2}$$

If n is even, $1 \pm n$ is odd and $B_n = 0$.

$$\therefore i_a = \frac{I_m}{\pi} + \frac{I_m \sin\theta}{2} + \frac{2\,I_m}{\pi}\left[\frac{1}{1 - n^2}\right]\cos n\theta$$

where n represents all the even numbers to infinity.

$$= \frac{I_m}{\pi} + \frac{I_m \sin\theta}{2} + \frac{2\,I_m}{\pi}\left[\frac{\cos 2\theta}{1 - 4} + \frac{\cos 4\theta}{1 - 16} + \frac{\cos 6\theta}{1 - 36} + \ldots\ldots\ldots\right]$$

$$\text{or } i_a = \frac{I_m}{\pi} + \frac{I_m \sin\theta}{2} - \frac{2\,I_m \cos 2\theta}{3\pi} - \frac{2\,I_m \cos 4\theta}{15\pi} - \ldots\ldots\ldots\ldots\ldots\ldots$$

END