TELEPHONE TRAFFIC

CONTENTS

INTRODUCTION

This pamphlet should be read in conjunction with E.P. Draft Series TELEPHONES 6/2 which deals in outline with the traffic variations over different time periods, the busy hour and the grade of service.

Telephone traffic is primarily composed of the calls originated by subscribers and, by definition, is measured by the product of the number of calls passing over the group of circuits, or trunks, under consideration and their average duration. There is no named unit for this aspect of telephone traffic, the numerical value obtained merely has the dimensions of calls and time, e.g. 6 call minutes or 0.1 call hours. The number of trunks provided between any two points must however, be sufficient to handle the average number of simultaneous calls which will be in progress during the busiest time of the day. For planning purposes, therefore, it is desirable to have a measure of telephone traffic which gives directly the average number of simultaneous calls in progress at any specified time. This pamphlet shows how such a unit is derived and used in conjunction with Erlang's formula to determine the number of trunks required to handle a given amount of traffic. Only groups of trunks working under full availability conditions are considered. Full availability means that all sources have access to all the trunks in the group, and in practice such a condition does not always exist. E.P. Draft Series TELEPHONES 6/4 deals with groups of circuits not working under full availability conditions.

THE UNIT OF TRAFFIC FLOW

The average number of simultaneous calls in progress during a period is known as the traffic flow, or intensity, and is measured directly in units known as 'erlangs'. It should be noted that originally the 'erlang' was known as the 'traffic unit' or 'T.U.' The unit can be derived from observations of calls and their duration over a specified period, and provides directly the following information:-

(a) the average number of simultaneous calls during the specified period,

(b) the portion of the specified period for which a circuit is occupied, and

(c) the number of calls which originate during a period equal to the average holding time of the calls occurring in the specified period.

The traffic flow, A erlangs, during a period of time T, can be determined from observations of the number of calls, C, and their average duration t, when T and t are expressed in the same units, as follows:-

By definition the volume of traffic during the period T is equal to the number of calls, C, times their average duration, t. Thus

$$\text{Volume of traffic} = C \times t$$

By applying simple reasoning the volume of traffic is equal also to the product of T and the average number of simultaneous calls, A, during the period T. Thus

$$\text{Volume of traffic} = T \times A$$

Hence

$$C \times t = T \times A$$

$$\therefore \quad A = \frac{Ct}{T} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

The equation (1) expresses the definition of traffic flow which has been adopted by the International Telecommunication Union, I.T.U., and is as follows:-

The average traffic intensity during a period T on a group of circuits, or switches, is the sum of the holding times divided by T, these holding times and the period of T, of course, being expressed in the same units.

Hence, if t is expressed as a fraction of the period T, and T considered as unity then

$$A = Ct$$

Because Ct is a measure of the volume of traffic, the expression A = Ct often leads to the erroneous statement 'x erlangs of traffic'; the numerical value of x in such a statement is only of value for planning purposes if the period during which the C calls occur is known. For planning purposes the B.P.O. use the busiest hour of the day, the busy hour, as the period T, and unless stated otherwise 'erlangs of traffic' refer to a period of one hour. Thus 6 erlangs of traffic means an average traffic intensity of 6 simultaneous calls during the busy hour.

The use of one hour as the period T leads to the definition 'one erlang is equal to the traffic intensity on one circuit fully occupied for one hour'. Thus the number of erlangs indicates the portion of the hour that a circuit is engaged, or occupied, e.g. if the average traffic intensity on a circuit is 0.6 erlang, the circuit is engaged for 36 minutes of the hour concerned. It follows that when the traffic is greater than 1 erlang, more than 1 circuit will be occupied, e.g. an average traffic intensity of 3.43 erlang will fully occupy 3.43 circuits for the hour. In practice, however, in order to provide a good grade of service 10 circuits are required to handle 3.43 erlang.

Finally, the definition that 'the average traffic intensity in erlangs is numerically equal to the average number of calls which originate during the average holding time of the calls' can be illustrated as follows:-

C calls originate in the period T, and the average holding time as a fraction of T is t; thus the average number of calls which originate during t is equal to Ct, which has already been shown to equal traffic intensity in erlangs.

The foregoing definitions of the erlang are illustrated by the following simple examples.

1. The number of calls carried by a group of circuits is counted at intervals of 10 minutes during the hour and the average holding time is 10 minutes. The number of calls in progress simultaneously were 12, 13, 10, 15, 10 and 12.

(a) Intensity of traffic:

$$A = \frac{Ct}{T}$$

$$= \frac{(12 + 13 + 10 + 15 + 10 + 12)10}{60}$$

$$= \frac{720}{60}$$

$$= 12 \text{ erlangs.}$$

(b) The average number of calls originating during the average holding time:

A total of 72 calls are originated in the hour, therefore the average number originated during the average holding time 10 minutes, is

$$\frac{72}{60} \times 10 = 12,$$ i.e. numerically equal to the traffic intensity.

(c) The circuit occupancy:

The volume of traffic is equal to Ct, that is 72 $\times$ 10 minutes which is equal to 12 call hours. The traffic is such, therefore that 12 circuits would be fully occupied for the hour.

2. During a particular busy hour a group of circuits carried 200 calls each of 3.0 minutes duration.

(a) Intensity of traffic:

$$A = \frac{Ct}{T}$$

$$= \frac{200 \times \frac{3}{60}}{1}$$

$$= 10 \text{ erlangs}$$

(b) The average number of calls originating during the average holding time:

200 calls are originated in 60 minutes, therefore the number originated in 3 minutes

$$= \frac{200}{60} \times 3$$

$$= 10$$

(c) The average number of calls in progress simultaneously:

200 calls each of 3.0 minutes duration occur in 60 minutes, the average number of calls in progress simultaneously

$$= \frac{200 \times 3}{60}$$

$$= 10$$

If the average number of calls in progress simultaneously is 10 it follows that, theoretically, 10 circuits would be fully occupied for the hour.

MEASUREMENT OF TRAFFIC FLOW

In practice, information about the traffic flow is obtained either in terms of the total number of calls during the busy hour and their average duration, or of the average number of simultaneous calls in progress during the hour.

The holding times of automatic equipment differ widely because of the short seizures which result from handsets being knocked accidentally from their telephones, or abandonment of calls before the complete number has been dialled. Such seizures ar common occurrences and contribute considerably to the occupancy of the earlier ranks of selectors, consequently the number of calls times average duration method of determining traffic flow is not used in automatic exchanges.

The average number of calls in progress during the hour gives the traffic flow directly in erlangs, and can easily be obtained by counting directly and at frequent intervals the number of calls simultaneously in progress. The average of the results gives the number of erlangs, and the accuracy is greater the shorter the interval between counts. Counting can be

(a) by visual observation of the number of switches or circuits, in a group, which are engaged.

(b) by manual testing of each circuit in the group.

(c) by automatic recording on meters.

If a route is provided with circuits on the correct basis, the tendency will be for most of the circuits to be in use during the busy period. This suggests that, insteac of counting the engaged circuits, it would normally be quicker to count the idle circuits, and this is in fact done for records taken manually on circuits from switchboards. Most large exchanges are fitted with automatic traffic recorders, whicl have the advantage of being able to test over a large number of circuits on many grouן in each cycle of tests.

The function of the automatic traffic recorder is to test for engaged conditions over all the circuits in a number of groups, and to record on a number of meters ever; engaged condition found. The tests are normally made once every 30 seconds and take place over a 1½ hour period; meter readings are taken at half-hourly intervals and th busy hour is the greater of the two 1 hour periods so measured.

The average number of simultaneous calls in a group	=	Total readings in 1 hour for all the circuits in the group	÷	Number of test cycl

Therefore

$$\text{Traffic flow in erlangs} = \frac{\text{Total readings in 1 hour period}}{120}$$

Example

The initial reading of the meter connected to a certain group of circuits is 0172, and the second, third and fourth readings are 0434, 0622 and 0848 respectively.

The readings are tabulated and the number of engaged conditions found in the two 1 hour periods are obtained as shown.

3rd reading	0622	4th reading	0848
1st reading	0172	2nd reading	0434
1 hour difference	0450	1 hour difference	0410

The busy hour is, therefore, the first 1 hour period and the traffic flow in that period

$$= \frac{450}{120}$$

$$= 3.75 \text{ erlangs}$$

TYPES OF TRAFFIC

PURE CHANCE TRAFFIC

The number of calls in progress varies from one instant to the next, the actual number depending on the time of origin and the duration of each call. In practice, although not theoretically correct, the traffic originated by subscribers is assumed to be of a pure chance nature.

Pure chance traffic is defined as traffic in which a call is as likely to originate at one moment as at any other; this implies that the number of sources from which calls originate is infinite. In practice the number of sources is always finite and if all the sources originated calls simultaneously there would be no liklihood of any further calls occurring. However, the proportion of the number of sources to the number of calls in progress at any one time is usually very small, consequently it is reasonable to assume that subscribers' traffic is originated on a basis of pure chance.

SMOOTH TRAFFIC

Where the traffic on a group of circuits does not at any time differ greatly from the average traffic measured over a period of time, it is said to be 'smooth'. Smooth traffic conditions apply

(a) when the number of sources from which the calls originate is small and the traffic large, since in such circumstances the chance of a further call being originated diminishes as the number of calls in progress grows larger.

(b) at a particular stage in a series of stages when the peaks of the originating traffic have been spread over a number of groups of circuits by the inter-connexion arrangements.

The smoother the traffic the fewer the number of circuits required to deal with a given volume of traffic at the same grade of service.

PROBABILITY AND GRADE OF SERVICE

The assumed pure chance nature of telephone traffic allows the theories of probability to be applied to the problem of finding the number of calls lost when a number of trunks are offered a given number of erlangs. A Danish engineer, A. K. Erlang, has evolved a mathematical relationship between the quantities, and although it makes certain assumptions it has been considered sufficiently valid for certain practical applications. Before discussing Erlang's formula some of the simpler aspects of probability and their application to telephone traffic will be considered.

PROBABILITY

The occurrence of an event that is as likely to happen as not can be expressed as a probability and given a numerical value equal to the ratio

$$\frac{\text{the number of times the event is possible}}{\text{the total number of possible occurrences}}$$

When the number of times the event is possible equals the total number possible occurrences, the happening of the event is a certainty and the numerical value of the probability is 1. If the event is an impossibility the value of the probability is 0, consequently all values of probability fall between 0 and 1.

It is a certainty that an event will either happen or fail to happen, thus if P is the probability of the happening of an event, the probability of its failure to happen is 1–P.

Single circuits

Let a circuit which is engaged be considered as an event, then if 3 circuits in a group of 10 are engaged, the probability of ONE circuit being found engaged at a random choice is $\frac{3}{10}$, and if 9 circuits are engaged the probability is $\frac{9}{10}$. A circuit must be either free or engaged, thus if 3 circuits are engaged the probability of finding ONE circuit free is $(1 - \frac{3}{10}) = \frac{7}{10}$.

If a circuit is engaged for x minutes in an hour, the probability of finding the circuit engaged during the hour can be expressed as

$$\frac{\text{number of minutes circuit engaged}}{\text{number of minutes in the hour}} = \frac{x}{60}$$

It is very important to note that the fraction $\frac{x}{60}$ is numerically equal to the traffic in erlangs carried by a circuit which is engaged for x minutes in the busy hour. Thus the traffic in erlangs carried by a SINGLE circuit is numerically equal to the probability of finding the circuit engaged during the hour.

Groups of circuits

When two circuits which are just as likely to be free as engaged, are tested simultaneously the probability that they both test engaged can be determined as follows:-

There are 4 possible results to the simultaneous test as shown in the table

Circuit 1	Free	Engaged	Free	Engaged
Circuit 2	Free	Free	Engaged	Engaged

Thus the probability that both circuits will test engaged is $\frac{1}{4}$ or 0.25. It is important to note that the probability that both circuits will test free is also 0.25, but that one will test free and the other engaged is 0.5. If, however, consideration is paid to which circuit is free the probability is 0.25.

If the separate circuits each have a given probability of being found engaged, the probability that both will be engaged simultaneously is equal to the product of the separate probabilities. The foregoing follows from the theorem that 'the probability of the simultaneous or successive occurrence of a number of events is the product of the probabilities of the separate occurrences'.

Let the probability of each circuit being found engaged be 0.4, then the probability of finding both circuits engaged simultaneously is

$$P = 0.4 \times 0.4 = 0.16$$

and the probability of finding both circuits free simultaneously is

$$P = (1 - 0.4) \times (1 - 0.4) = 0.36$$

The probability of finding circuit 1 engaged is 0.4 and of finding circuit 2 free is 0.6, hence the probability of finding circuit 1 engaged and circuit 2 free is

$$P = 0.4 \times 0.6 = 0.24,$$

in a similar fashion the probability of finding circuit 1 free and circuit 2 engaged is also 0.24. The two circuits can only exist in one of the four states already considered, hence the sum of the probabilities 0.16, 0.36, 0.24 and 0.24 equals 1, i.e. certainty.

When consideration is not given to which circuit is free in determining the probability of one circuit free and the other engaged, the theorem that, 'the probability of the occurrence of either one or the other of two mutually exclusive events is equal to the sum of the probabilities of the single events', is applied. The event is one specific circuit engaged and the other free, and the events are mutually exclusive because the occurrence of one of them prevents the occurrence of the other. Hence the probability of finding one circuit engaged and the other free is

$$P = 0.24 + 0.24 = 0.48$$

The foregoing method of working can be applied to groups containing more than 2 circuits. The number of possible results to a simultaneous test of x circuits is equal to 2^x, thus when a group of 4 circuits is tested there are 16 possible results.

Let each circuit of a 4 circuit group have a probability of being engaged of 0.02. The probability of any particular result to the test is the product of the probabilities of the separate circuit conditions, hence

(a) the probability of all circuits being engaged $= .02 \times .02 \times .02 \times .02$

$= 16 \times 10^{-8}$

(b) the probability of all circuits being free $= .98 \times .98 \times .98 \times .98$

$= .92237$

(c) the probability of the test cct. 1 free, cct. 2 engaged cct. 3 engaged and cct. 4 free $= .98 \times .02 \times .02 \times .98$

$= .000384$

(d) the probability of a test in which at least one circuit is engaged $= 1 - .92237$

$= 0.07763$

It should be noted that the probability (d) is very nearly equal to the sum of the probabilities of the separate circuits being engaged; this point is dealt with fully later in this pamphlet.

GRADE OF SERVICE

The traffic intensity in erlangs is the average intensity during the hour, therefore the intensity at any instant during the hour is likely to be more or less than the number of erlangs. If, therefore, the traffic estimated to be offered to a group of circuits is, say, 8 erlangs, and only 8 circuits are provided, it is a certainty that some of the offered traffic will be lost. The certainty will be reduced to a probability if more than 8 circuits are provided, the greater the number of circuits the lower the probability.

The probability of a call being lost can be expressed by the ratio

$$p = \frac{\text{calls lost}}{\text{calls offered}}$$

and consequently when the calls are considered to have equal holding times

$$p = \frac{\text{traffic lost}}{\text{traffic offered}}$$

In practice sufficient circuits are provided to give a specified value to the probability of a call being lost during the busy hour; the probability in this application is known as the 'grade of service'. Details of the values of the grades of service for various types of circuits are given in E.P. Draft Series, TELEPHONES 6/2.

The grade of service, B, is equal to the ratio of the traffic lost to the traffic offered, it must therefore also be equal to the probability of finding all the circuits in the group simultaneously engaged, that is

$$B = \frac{\text{number of minutes all circuits simultaneously engaged}}{\text{number of minutes in the hour}}$$

Thus if B = .00625, i.e. $\frac{1}{160}$, then 1 call in 160 is lost and all circuits are simultaneously engaged for a total period of 22.5 seconds in the busy hour. It follows that when only 1 circuit is concerned, the traffic carried in erlangs is also numerically equal to the grade of service, e.g. if the circuit carries .75 erlang the circuit is occupied for .75 of the hour, hence the grade of service is .75.

The grade of service quoted for the purpose of equipment provision is based on the traffic during the busy hour, consequently at times other than the busy hour it is governed by the ratio of the traffic in the hour under consideration to the traffic in the busy hour.

It should be noted that as the value of B increases the grade of service in terms of calls lost worsens, e.g.

B = .001 = 1 call lost in 1000, and

B = .002 = 1 call lost in 500.

Overall grade of service

In a telephone exchange network the ranks of selectors and junction groups each have a specified grade of service, consequently the overall grade of service on a subscriber to subscriber call is dependent both on the value of B at each link in the call and the number of links involved.

Consider, for simplicity, the circuit to consist of 2 links having grades of service of .005 and .02 respectively. The links can exist only in any one of the four following states

	1	2	3	4
Link 1 Link 2	Free Free	Free Engaged	Engaged Free	Engaged Engaged

From the theorem that 'the probability of the successive occurrence of a number of events is the product of the probabilities of the separate occurrences', it follows that,

the probability of condition 1 is	$(1 - .005) \times (1 - .02)$	=	.9751
" " " " 2 "	$(1 - .005) \times (.02)$	=	.0199
" " " " 3 "	$(.005) \times (1 - .02)$	=	.0049
" " " " 4 "	$(.005) \times (.02)$	=	.0001

Thus for a call to be successful the two links must both be free; therefore the probability of a call being successful is

$$(1 - .005) \times (1 - .02) = .9751$$

For a call to fail the two links must be in one of the conditions 2, 3 or 4; these events are mutually exclusive, therefore the probability of the occurrence of one of them is the sum of their separate probabilities. Thus the probability of a call failing is

$$.0199 + .0049 + .0001 = .0249$$

This value is very nearly equal to the sum of the two separate grades of service, i.e. $.005 + .02 = .025$.

The problem may also be resolved in the following way:-

Consider the overall grade of service for a local call on a hypothetical exchange having two ranks of selectors, the 1st selectors having a grade of service, B_1, of .005 and the final selectors a grade of service, B_2, of .02.

Let the traffic offered to the 1st selectors = A erlangs, then

$$\begin{aligned}
\text{traffic lost at 1st selectors} &= B_1 A \\
\therefore \text{ traffic offered to final selectors} &= A - B_1 A \\
\text{traffic lost at final selectors} &= B_2(A - B_1 A) \\
\therefore \text{ successful traffic} &= (A - B_1 A) - B_2(A - B_1 A) \\
&= A - AB_1 - AB_2 + AB_1 B_2 \\
\therefore \text{ traffic lost overall} &= A - (A - AB_1 - AB_2 + AB_1 B_2) \\
&= AB_1 + AB_2 - AB_1 B_2 \\
&= A(B_1 + B_2 - B_1 B_2) \\
\therefore \text{ overall grade of service } B &= B_1 + B_2 - B_1 B_2
\end{aligned}$$

Substituting the values given for B_1 and B_2

$$\begin{aligned}
B &= .005 + .02 - (.005 \times .02) \\
&= .005 + .02 - .0001 \\
B &= .0249
\end{aligned}$$

The value for the overall grade of service is the same by both methods of working and is very slightly less than the sum of the individual values; this is to be expected when sm... values of B are involved because the traffic passed from one rank of selectors to the next is very nearly the same as that offered to the particular rank.

The sum of the individual grades of service is accurate to within approximately 2% for a grade of service of .02 at three successive links, and 3% for a grade of service of .033 also at three successive links. The error increases with the number of links, being 10% for five links when the grade of service is .02. In the event of calculations based on lower grades of service than those already considered, the general formula

$$\text{Overall grade of service} = 1 - (1 - B)^N$$

where B is the grade of service at each of N links, is used.

Summarizing the foregoing.

(a) The probability of a call succeeding over a number of switching stages and/or junctions is the product of the individual probabilities of success.

(b) The overall grade of service over a number of switching stages and/or junctions may be taken as the sum of the individual grades of service when the individual values are small.

In practice the busy hour traffic varies from day to day, consequently the grade of service at any switching stage or junction route is based on the average of serveral busy hours. Also the busy hours of the various switching stages and junctions involved in a call may occur at different times, therefore the overall grade of service is likely to be better than the sum of the individual values.

Measurement of grade of service

In automatic exchanges meters are provided by means of which it is possible to keep a continuous check on the grade of service given on the various groups of circuits. The meters are in addition to the automatic traffic recorder which normally is used only to provide complete records of the traffic at 6-monthly intervals.

The chief types of meter provided for the supervision of the grade of service are:-

(a) overflow meters,

(b) late-choice call meters, commonly termed congestion meters, (LCCM)

(c) late-choice traffic unit meters, commonly termed congestion time-unit meters, (LCUM)

(d) group occupancy time meters, (GOTM) and

(e) call-counting meters.

Overflow meters. An overflow meter is connected such that it records very nearly the number of calls which occur when all the circuits in the group are engaged. In practice, when all the circuits on a selector level are engaged and another call occurs, the wipers of the particular selector step to the 11th outlet and cause the overflow meter to operate and remain operated until the selector is released by the calling subscriber. In non-director areas such calls are held for approximately 10 seconds, and in director areas for 15 seconds. If the wipers of another selector step to the 11th outlet the event will not be recorded because the other selector is holding the meter operated; however, if the total number of 'overflow calls' is small the resultant error can be neglected.

A value known as the 'critical reading' obtained from the overflow meter readings which, if consistently exceeded will indicate that the grade of service has fallen below a certain figure, can be determined as follows:-

The number of circuits in a group are provided so that a particular grade of service is given when the group is offered a specific number of erlangs. The traffic lost, or overflowing, from the group is given by the product of the traffic offered, A, and the grade of service, B,

i.e. Overflow traffic $= BA$

By definition A is equal to the number of calls originating during the average holding time, t, hence

$$\text{Number of lost calls} = \frac{BAT}{t} \quad \text{where } T = \text{total period of record, and}$$
$$t = \text{average holding time in terms of } T$$

Hence, as the period T is the busy hour and t is in hours,

$$\text{Number of lost calls} = \frac{BA}{t}$$

and if the number of calls lost as recorded on the meter exceeds this figure the actual grade of service on the group is deteriorating. Consequently the numerical value given by the expression BA/t is known as the critical figure.

It should be noted that in determining the critical figure it is assumed that both effective and lost calls have the same duration, t; in practice the resultant error is negligible when the grade of service is good, i.e. the value of B is small. In practice, when B is better than 1 in 50 the traffic offered may be considered to be equal to the traffic carried, thereby simplifying calculations.

Example

Find the critical overflow figure for a group of 40 trunks so arranged that they can carry 21.63 erlangs for a grade of service of .005; the average duration of the calls is 3.0 minutes.

$$\text{Critical value} = \frac{BA}{t}$$
$$= \frac{.005 \times 21.63 \times 60}{3}$$
$$= \underline{2.16 \text{ Answer}}$$

Late choice call meters and late choice traffic unit meters. These meters are provided on groups of circuits from continuously hunting switches such as subscribers uniselectors, where the overflow traffic cannot be measured. The LCCM is connected usually to the last circuit and records the number of times this circuit is seized. A contact of the LCCM completes a circuit for the LCUM which records the cumulative time that the circuit is in use, by being stepped once every half-minute. The reading of the LCUM gives the number of half-minutes the lost circuit is occupied, hence in the busy hour

$$\frac{\text{LCUM reading}}{120} = \text{Traffic in erlangs carried by the outlet}$$

The traffic carried by the last circuit in a group, for a given grade of service can be calculated as shown later in this pamphlet; the critical figure for the LCUM is, therefore, equal to the reading which represents this traffic.

Hence, critical figure $=$ Last circuit traffic at the specified grade of service $\times$ 120

Group occupancy time meters. These meters are used to record the cumulative time, usually in 1 second or 2 second intervals, during which all the circuits in a group are busy. Such a meter is fitted to each primary finder group in the linefinder system and records the time directly in seconds during which all the selectors in the group are engaged.

The proportion of the hour during which all the trunks are engaged is numerically equal to the grade of service for the group. If, therefore the grade of service is .005, the critical time for the group is .005 of an hour, i.e. 18 seconds. The critical figure for the meter depends on the time pulse employed; if a 2 second time pulse is used the critical time of 18 seconds is, of course, 9.

Call-counting meters. These meters are connected to certain types of equipment such as directors and senders, and operate once for each completed call. The time for the particular piece of equipment to perform its function is known, hence the product of this time, in terms of an hour, and the meter reading over the period of an hour gives the traffic in erlangs carried.

The circuit arrangements of the meters discussed in this section are dealt with i the pamphlet on traffic recording.

ERLANG'S FORMULA

The mathematical relation between traffic intensity, grade of service and a given number of circuits which has been evolved by Erlang is based on the following assumptions:-

(a) that all the circuits in the group can be tested by every source of traffic; this means that full availability conditions must exist,

(b) that the traffic is of a pure chance nature; if the number of sources is in excess of 100 very little error results from this assumption,

(c) that all the calls which originate when all trunks are busy are lost, and that the holding time of these calls is zero,

(d) that the traffic is the average of a large number of busy hours, and that statistical equilibrium conditions exist, this implies that during a very short interval of time, dt, the total number of calls which terminate is equal on the average to the number which originate in the same period.

A method of deriving the relationship from first principles is given later in this pamphlet. The method is a version of that first produced by H.A. Longley and N. A. Hawkins, and published in the P.O.E.E.J. Vol. 41, Parts I and II 1948 under the heading "The Efficiency of Gradings".

Erlang's formula for the probability, P, of x circuits being engaged when A erlangs are offered to a group of N circuits is

$$P_x = \frac{\frac{A^x}{x!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} \cdots \frac{A^N}{N!}}$$

N! is known as 'factorial' N and is equal to $N \times (N-1) \times (N-2) \ldots \times (N-(N-1))$. By convention if $N = 0$, $N! = 1$.

The probability of all trunks being engaged is also the probability of a call failing, i.e. the grade of service, hence

$$B = \frac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} \cdots \frac{A^N}{N!}}$$

If it is desired to find the theoretical grade of service when a group of N trunks is offered a known value of A erlangs, it is only necessary to insert the values of N and A in the above formula and solve.

Example 1

A full availability group of 5 trunks is offered a busy hour average of 3 erlangs, what is

(a) the probability of 3 trunks being simultaneously engaged,

(b) the probability of all trunks being free, and

(c) the grade of service?

When a full availability group of N trunks is offered A erlangs of pure chance traffic the probability, P, of finding exactly x trunks engaged is given by

$$P_x = \frac{\frac{A^x}{x!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots \frac{A^N}{N!}}$$

(a) Thus the probability of finding 3 trunks engaged is

$$P_3 = \frac{\frac{3^3}{3!}}{1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}}$$

$$= \frac{\frac{27}{6}}{1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120}}$$

$$= \frac{\frac{27}{6}}{\frac{92}{5}}$$

$$= \frac{27 \times 5}{92 \times 6}$$

$$= \frac{45}{184}$$

$$= \underline{0.2445 \text{ Answer (a)}}$$

(b) The probability of finding all trunks free is the same as the probability of finding no trunks engaged.

$$P_0 = \frac{\frac{3^0}{0!}}{\frac{92}{5}}$$

$$= \frac{1}{\frac{92}{5}}$$

$$= \frac{5}{92}$$

$$= \underline{0.0543 \text{ Answer (b)}}$$

(c) The grade of service B,

$$B = \frac{\frac{3^5}{5!}}{\frac{92}{5}}$$

$$= \frac{\frac{243}{120}}{\frac{92}{5}}$$

$$= \frac{243 \times 5}{120 \times 92}$$

$$= \underline{0.1101 \text{ Answer (c)}}$$

Example 2

Calculate (a) the grade of service given, and (b) the traffic lost, by 5 switches arranged in a full availability group when offered 0.9 erlang.

(a) The grade of service $B = \dfrac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} \ldots \frac{A^N}{N!}}$

$$= \frac{\frac{0.9^5}{120}}{1 + 0.9 + \frac{0.9^2}{2} + \frac{0.9^3}{6} + \frac{0.9^4}{24} + \frac{0.9^5}{120}}$$

$$= \frac{0.0049}{2.45} \text{ approx.}$$

$$= .002$$

The grade of service is 0.002 Answer (a)

(b) The traffic lost = traffic offered $\times$ grade of service

$= 0.9 \times 0.002$

$= 0.0018$

The traffic lost is 0.0018 erlang Answer (b)

POISSON'S EXPRESSION

If the number of circuits in a group is large and the grade of service is to be good, a somewhat simpler expression can be used. The denominator of the Erlang formula approximates to the infinite series

$$1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} \ldots\ldots \frac{A^N}{N!} \ldots\ldots$$

as N becomes large. The sum of this series is equal to e^A, where e = base of Napierian logarithms i.e. 2.7183.

Substituting in Erlang's formula we have:-

$$B = \frac{A^N}{e^A N!}$$

This equation is known as Poisson's expression, and the results obtained by using it approximate closely to those obtained from the Erlang formula if N is large.

TRAFFIC CARRIED BY EACH TRUNK IN A GROUP

The circuits, or trunks, of a full availability group connected to the selector levels in an automatic exchange are always tested in the same order. The traffic offered and that carried by each trunk in such a group of N trunks offered A erlangs can be determined by use of Erlang's formula in the following way.

1ST TRUNK TRAFFIC

The trunks of the N circuit group are tested in the same order, thus the A erlangs may be considered to be first offered to a group consisting only of the 1st trunk, that is a 1 circuit group. The traffic mot carried by the 1st trunk is, however, not lost but is passed to the 2nd and subsequent trunks of the N circuit group. The traffic passed on from the 1st trunk is equal to the product of the traffi offered, A erlangs, and the grade of service, B_1, of the 1 circuit group when it is offered A erlangs. Thus

$$\text{Traffic passed on from 1st trunk} = AB_1$$

$$\text{By Erlang's formula, when } N = 1$$

$$B_1 = \frac{A}{1 + A}$$

$$\therefore \text{ traffic passed on from 1st trunk} = A \times \frac{A}{1 + A}$$

$$= \frac{A^2}{1 + A}$$

$$\text{The traffic carried by the 1st trunk} = A - \frac{A^2}{1 + A}$$

$$= \frac{A + A^2 - A^2}{1 + A}$$

$$= \frac{A}{1 + A}$$

$$\therefore \underline{\text{Traffic carried by 1st trunk}} = \frac{A}{1 + A}$$

This confirms the earlier statement that the traffic carried by a 1 circuit group is numerically equal to the grade of service.

2ND TRUNK TRAFFIC

The traffic carried by, and passed on from the 2nd trunk to subsequent trunks can be determined by considering the A erlangs to be offered to a 2 circuit group. The traffic not carried by the 2 circuit group is the product of A erlangs and the grade of service, B_2, of the 2 circuit group when offered A erlangs; it is this traffic whic is passed on to the subsequent trunks of the N circuit group. Thus the traffic passed on by the 2 circuit group can be expressed as

Traffic passed on by the 2 circuit group $= AB_2$

By Erlang's formula when $N = 2$

$$B_2 = \frac{\frac{A^2}{2!}}{1 + A + \frac{A^2}{2!}}$$

$$\therefore \text{Traffic passed on by the 2 circuit group} = A \times \frac{\frac{A^2}{2!}}{1 + A + \frac{A^2}{2!}}$$

The traffic carried by the 2nd trunk of the N circuit group is equal to the difference between the traffic passed on from the 1st trunk and that passed on from the 2nd trunk. Thus, traffic carried by the 2nd trunk $= B_1 A - B_2 A$

$$= A \times \frac{A}{1 + A} - A \times \frac{\frac{A^2}{2!}}{1 + A + \frac{A^2}{2!}}$$

$$\therefore \text{Traffic carried by 2nd trunk} = A \left(\frac{A}{1 + A} - \frac{\frac{A^2}{2!}}{1 + A + \frac{A^2}{2!}} \right)$$

The traffic carried by and passed on by any trunk in the N circuit group can be determined by applying the method used to find the traffic carried by, and passed on from, the 2nd circuit.

LAST TRUNK TRAFFIC

When the grade of service, B_N, of the N circuit group is known, the approximate traffic carried by the last trunk is given by the simple expression

$$\text{Last trunk traffic} = B_N \, (N - A)$$

The expression is derived as follows:-

From previous working it follows that the traffic carried by the Nth trunk is equal to

$$AB_{(N-1)} - AB_N$$

that is, the difference between the traffic passed on from the (N-1)th trunk and that lost from the Nth trunk.

The grade of service for the N circuit group,

$$B_N = \frac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \dots \frac{A^{(N-1)}}{(N-1)!} + \frac{A^N}{N!}}$$

Let the denominator of this expression equal K

$$\therefore B_N = \frac{\frac{A^N}{N!}}{K}$$

$$\therefore K = \frac{A^N}{N!} \times \frac{1}{B_N}$$

The grade of service for the (N-1) circuit group,

$$B_{(N-1)} = \frac{\frac{A^{(N-1)}}{(N-1)!}}{1 + A + \frac{A^2}{2!} + \ldots . \frac{A^{(N-1)}}{(N-1)!}}$$

$$\therefore B_{(N-1)} = \frac{\frac{A^{(N-1)}}{(N-1)!}}{K - \frac{A^N}{N!}}$$

$$= \frac{\frac{A^{(N-1)}}{(N-1)!}}{\left(\frac{A^N}{N!} \times \frac{1}{B_N}\right) - \frac{A^N}{N!}}$$

Divide through by $\frac{A^N}{N!}$

$$B_{(N-1)} = \frac{\frac{N}{A}}{\frac{1}{B_N} - 1}$$

$$= \frac{\frac{N}{A}}{\frac{1 - B_N}{B_N}}$$

$$= \frac{B_N \; N}{A(1-B_N)}$$

The value of B is usually small, and consequently only a very small error results from assuming that 1-B is equal to unity, hence it may be stated that

$$B_{(N-1)} = \frac{B_N \; N}{A}$$

Thus, substituting for $B_{(N-1)}$

$$\text{Traffic carried by Nth trunk} = AB_{(N-1)} - AB_N$$

$$= \frac{A.B_N \; N}{A} - AB_N$$

$$= B_N \; N - AB_N$$

$$\therefore \underline{\text{Last trunk traffic}} = B_N \; (N - A)$$

Example

Calculate the traffic carried by each of the first 3 trunks of a full availability group when offered 2 erlangs.

$$\text{Traffic carried by 1st trunk} = \frac{A}{1 + A}$$

$$= \frac{2}{1 + 2} = 0.667 \text{ erlangs.}$$

$$\text{Traffic carried by 2nd trunk} = \text{trunk offered} - \text{traffic passed on}$$

$$= \left(\frac{A}{1 + A} \times A\right) - \left(\frac{\frac{A^2}{2!}}{1 + A + \frac{A^2}{2!}} \times A\right)$$

$$= \left(\frac{2}{3} \times 2\right) - \left(\frac{\frac{4}{2}}{1 + 2 + \frac{4}{2}} \times 2\right)$$

$$= 1\frac{1}{3} - \frac{4}{5}$$

$$= \underline{0.533 \text{ erlangs}}$$

$$\text{Traffic carried by 3rd trunk} = \text{traffic offered} - \text{traffic passed on}$$

$$= \frac{4}{5} - \left(\frac{\frac{2^3}{2 \times 3}}{1 + 2 + \frac{4}{2} + \frac{8}{6}} \times 2\right)$$

$$= 0.8 - 0.421$$

$$= \underline{0.379 \text{ erlangs}}$$

$$\left.\begin{array}{l} \text{The traffic carried by 1st trunk} = 0.667 \text{ erlangs} \\ \text{" " " " " "} = 0.533 \text{ erlangs} \\ \text{" " " " " "} = 0.379 \text{ erlangs} \end{array}\right\} \underline{\text{Answer}}$$

The sum total of the traffic carried by the first 3 trunks in the previous example is 1.579 erlang; the remaining 0.421 erlang is passed to the 4th and subsequent trunks in the group. If, however, the group had consisted only of the 3 trunks the 0.421 erlang will be lost, and consequently the grade of service, B, equals $\frac{0.421}{2} = .2105$.

The result obtained in this fashion for the grade of service can be checked by the use of Erlang's formula

$$B = \frac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}}$$

$$= \frac{\frac{2^3}{3!}}{1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!}}$$

$$= \frac{\frac{8}{6}}{1 + 2 + \frac{4}{2} + \frac{8}{6}}$$

$$= \frac{\frac{8}{6}}{\frac{38}{6}}$$

$$= \frac{8}{38}$$

$$\therefore B = \text{very nearly } .2105$$

TRAFFIC TABLES

Whilst it is possible to obtain values for the traffic offered to a trunk and for traffic passed on or lost by that trunk in the manner described in the foregoing section, a large amount of laborious work is involved in obtaining answers for individual cases. In practice the quantities of traffic carried by particular trunks, or passed on from particular trunks, are tabulated from families of curves which have been derived by plotting quantities of traffic determined in the manner already described.

By using the curves shown in Figs. 1 and 2 (appended) it is possible to determine the traffic carried by each choice, or trunk, of a full availability group of up to 10 trunks when offered a known quantity of traffic, by subtracting the value of traffic offered to the next choice.

Example

Traffic carried by first choice = Traffic offered to 1st choice − Traffic offered to 2nd choice

Assuming 5.0 erlangs offered to the 1st choice, then from Fig. 1

Traffic carried by 1st choice = 5.0 - 4.15 = 0.85 erlang

Similarly,

Traffic carried by 3rd choice = Traffic offered to 3rd choice − Traffic offered to 4th choice

From Fig. 1,

Traffic carried by 3rd choice = 3.375 - 2.65 = 0.725 erlang.

The traffic carried by a group of 10 trunks when offered 5 erlangs is found as follows:-

Traffic carried by group of 10 circuits = Traffic offered to the group − Traffic passed on from group

The traffic passed on will be equal to that offered to an imaginary 11th trunk. From Fig. 2, the 11th choice of a full availability group which is offered 5 erlangs will be offered 0.092 erlang. Thus a full availability group of 10 trunks will carry

$$5.0 - 0.092 = \underline{4.908 \text{ erlang}}$$

The grade of service for such a group of circuits when offered 5.0 erlangs can be found as follows:-

$$\text{Grade of service} = \frac{\text{traffic lost from group}}{\text{traffic offered to group}}$$

$$= \frac{0.092}{5.0}$$

$$= \underline{0.0184}$$

The traffic capacity at a specified grade of service of various groups of trunks, assuming full availability conditions, is given in Table 1.

No. of trunks	Capacity in erlangs for grade of service of:-		
	0.002	0.005	0.01
1	0.002	0.005	0.01
2	0.065	0.105	0.15
3	0.25	0.35	0.45
4	0.53	0.70	0.87
5	0.90	1.13	1.36
6	1.32	1.62	1.88
7	1.80	2.16	2.47
8	2.31	2.73	3.11
9	2.85	3.33	3.77
10	3.43	3.96	4.46
11	4.02	4.61	5.1
12	4.63	5.28	5.9
13	5.27	5.97	6.6
14	5.92	6.63	7.3
15	6.58	7.38	8.1
16	7.26	8.10	8.9
17	7.95	8.84	9.7
18	8.64	9.58	10.5
19	9.35	10.34	11.3
20	10.07	11.10	12.0
40	25.7	27.3	29.0
70	51.0	53.7	56.3
100	76.4	80.9	84.1

TABLE 1

For a given grade of service it can be seen from the table that the average traffic per trunk is greater with a large group of trunks than with a small group. At a grade of service of 0.002, a group of 10 trunks can carry 3.43 erlangs, an average of 0.343 erlangs per trunk; thus on average each trunk is engaged for 0.343, or approximately 20½ minutes, of the hour. At the same grade of service 20 trunks can carry 10.07 erlangs, an average of 0.5035 erlangs per trunk; thus on average each trunk is engaged for approximately 30 minutes of the hour. Therefore from a traffic-carrying point of view large groups of trunks are more efficient than small groups, but the capacity of large groups to carry overloads of traffic without serious deterioration of the grade of service is less than that of small groups, because their trunks are on average occupied for a greater proportion of the hour.

THE DERIVATION OF ERLANG'S FULL AVAILABILITY FORMULA

The assumptions underlying the derivation of the Erlang formula have already been stated, but for completeness are again given, and are as follows

(a) that full availability conditions obtain.

(b) that traffic is of a pure chance nature.

(c) that the calls which originate when all trunks are busy are lost, and that the holding time of these calls is zero,

(d) that the traffic is the average of a large number of busy hours, and

(e) that statistical equilibrium conditions exist. This implies that during a very short interval of time, dt, the total number of calls which terminate is equal on the average to the number which originate in the period dt.

This derivation, which was produced by H. A. Longley and N. A. Hawkins, and published in the P.O.E.E.J. Vol. 41 Parts I and II, is based on the examination of a group of circuits which is in statistical equilibrium and accepting A erlangs in a random manner.

Consider, for simplicity, a group of 3 circuits which are always tested in the same order, that is circuit 1, circuit 2 and then circuit 3. The group must, at any instant, be in one of the eight states shown in Fig. 3; for convenience the eight states are subdivided into four classes, P_0, P_1, P_2 and P_3. The arrival or cessation of a call causes the condition existing at that instant to change; commencing with all circuits free, the trend of the change in circuit conditions is illustrated in the following four examples:-

(i) The initial condition is P_0, that is all circuits free; a call arriving must, because of the order of testing, change the condition to P_1a.

(ii) The arrival of another call changes condition P_1a to condition P_2a; the cessation of a call, however, reverts condition P_1a back to P_0.

(iii) A call arriving when condition P_2a exists creates condition P_3; the cessation of a call, however, reverts condition P_2a to condition P_1a or P_1b depending on the call which ceases.

(iv) A call arriving when condition P_3 exists is, by the assumptions underlying the derivation, lost and considered to have zero holding time; the cessation of a call creates one of the P_2 conditions, the actual condition depending on which call ceases.

A diagram showing how the conditions of three circuits change with the arrival and cessation of pure chance calls is given in Fig. 4. Consider the circuits to be in the condition P_1b, the arrival of a call changes the condition to condition P_2a as indicated by the solid line arrow; the cessation of the call on circuit 2 changes the condition to P_1a as indicated by the broken line arrow.

The proportion of any given period of time that each of the four main condition, P_0, P_1, P_2, and P_3, exist can be represented by the labelling used in Fig. 4, and the sum of these proportions will equal the period of time, hence

$$P_0 + P_1 + P_2 + P_3 = 1 \qquad \text{where } 1 = \text{the period of time}$$

and

$$P_0 + (P_1a + P_1b + P_1c) + (P_2a + P_2b + P_2c) + P_3 = 1$$

The assumptions (b) and (d) imply that, over a period of time, each circuit condition will receive traffic with the same average intensity, A erlangs.

Consider the period of time to be equal to the average holding time of the calls. The average number of calls arriving in the holding time is equal to A, hence in the proportion of the period, say, P_2a the number of calls arriving is AP_2a, and in the period P_0 the number is AP_0. It should be remembered that assumption (c) means that calls arriving during the period P_3 are ignored.

On the average, any call existing on any circuit will terminate within the average holding time. Thus the average number of calls ceasing on any circuit during any proportion, P, of the average holding time will be P; this means that when the circuit condition P_1b is considered the average number of calls ceasing is P_1b, but when P_2b is considered the average number of calls ceasing is $2P_2b$ because the condition contains two engaged circuits.

When the system is in statiscal equilibrium the amount of traffic which tends to create a given condition equals the amount which tends to destroy that condition.

Example

Condition P_1a, Fig. 4, is created by the arrival of traffic during condition P_0, i.e. AP_0, and the cessation of traffic during conditions P_2a and P_2b, i.e. $P_2a + P_2b$. On the other hand the condition is destroyed by the arrival of traffic during P_1a, i.e. AP_1a, and the cessation of traffic during P_1a, i.e. P_1a.

$$\text{Thus} \qquad AP_0 + P_2a + P_2b = AP_1a + P_1a.$$

Erlang's full availability can be derived by extending the reasoning applied in the foregoing example.

Condition P_3, Fig. 4, is created only by the arrival of traffic during conditions P_2a, P_2b and P_2c, i.e. $A(P_2a + P_2b + P_2c)$. On the other hand the condition is destroyed by the cessation of a call on any one of the three circuits, i.e. $P_3 + P_3 + P_3 = 3P_3$.

$$\text{Thus} \qquad A(P_2a + P_2b + P_2c) = 3P_3$$

$$\therefore AP_2 = 3P_3$$

By referring to Fig. 4 basic equations for the group of three circuits can be formed, and are as follows:-

CONDITION	CREATIVE FACTORS = DESTRUCTIVE FACTORS	
P_0	$P_1 = AP_0$	(1)
P_1a	$AP_0 + P_2a + P_2b = AP_1a + P_1a$	
P_1b	$P_2a + P_2c = AP_1b + P_1b$	
P_1c	$P_2b + P_2c = AP_1c + P_1c$	
Thus P_1	$AP_0 + 2P_2 = AP_1 + P_1$	(2)
P_2a	$AP_1a + AP_1b + P_3 = AP_2a + 2P_2a$	
P_2b	$AP_1c + P_3 = AP_2b + 2P_2b$	
P_2c	$P_3 = AP_2c + 2P_2c$	
Thus P_2	$AP_1 + 3P_3 = AP_2 + 2P_2$	(3)
P_3	$AP_2 = 3P_3$	(4)

From the equations 1, 2, 3 and 4.

$$AP_0 = P_1 \quad \therefore P_1 = AP_0$$

$$AP_1 = 2P_2 \quad \therefore P_2 = \frac{A}{2}P_1 = \frac{A}{2} \times AP_0 = \frac{A^2}{2} P_0$$

$$AP_2 = 3P_3 \quad \therefore P_3 = \frac{A}{3}P_2 = \frac{A}{3} \times \frac{A^2}{2}P_0 = \frac{A^3}{3 \times 2} P_0$$

It has already been stated that

$$P_0 + P_1 + P_2 + P_3 = 1$$

$$\therefore P_0 + AP_0 + \frac{A^2}{2}P_0 + \frac{A^3}{3 \times 2} P_0 = 1$$

$$\therefore P_0 \left(1 + A + \frac{A^2}{2} + \frac{A^3}{3 \times 2}\right) = 1$$

$$\therefore P_0 = \frac{1}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}}$$

where 2! and 3! are factorial 2 and 3 respectively, factorial N, N!, evaluates to $N \times (N-1) \times (N-2) \ldots\ldots\ldots \times (N - (N-1))$.

The expression obtained for P_0 gives a numerical value to the proportion of the period of time that such a condition, i.e. all circuits free, exists. This condition, P_0, can also be considered as the proportion of the time during which none of the circuits are engaged, also the proportion of the time is numerically equal to the probability, hence the probability of no circuits being engaged when three circuits are offered A erlangs is given by

$$P_0 = \frac{1}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}}$$

In general terms, considering N circuits

$$P_0 = \frac{1}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} \cdots\cdots \frac{A^N}{N!}}$$

By similar reasoning, the probability of finding 3 circuits engaged can be found as follows:-

$$P_3 = \frac{A^3}{3!} \cdot P_0$$

$$= \frac{A^3}{3!} \times \frac{1}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}}$$

$$\therefore P_3 = \frac{\frac{A^3}{3!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}}$$

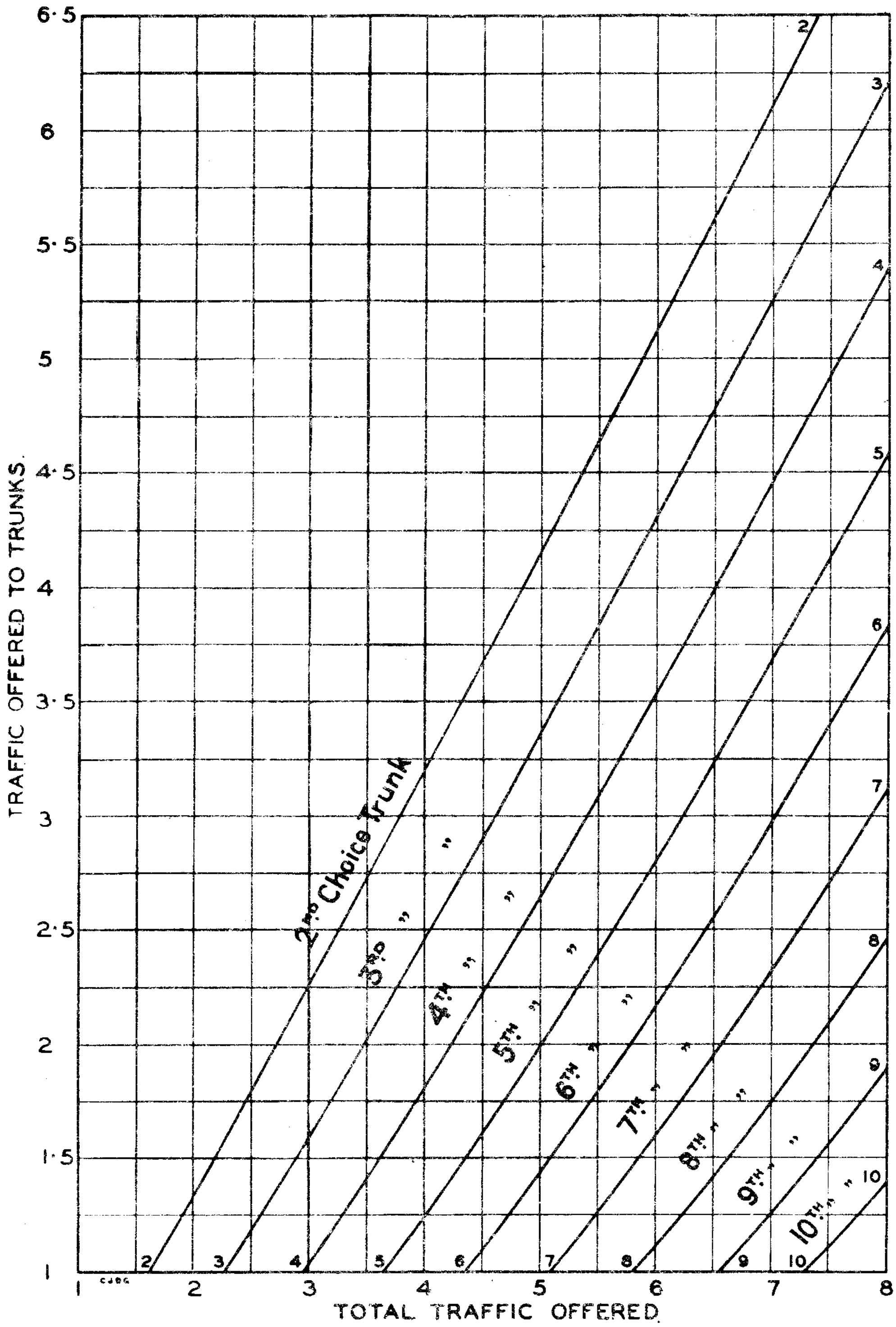
Again, in general terms when a group of N circuits is considered,

$$P_N = \frac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} \cdots\cdots + \frac{A^N}{N!}}$$

The probability of all circuits in a group being engaged is numerically equal to the grade of service, B, for the group. Thus the grade of service for a group of N circuit when offered A erlangs is

$$B = \frac{\frac{A^N}{N!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} \cdots\cdots \frac{A^N}{N!}}$$

END


2ND Choice Trunk
3RD "
4TH "
5TH "
6TH "
7TH "
8TH "
9TH "
10TH "
TRAFFIC OFFERED TO TRUNKS
TOTAL TRAFFIC OFFERED
1
1·5
2
2·5
3
3·5
4
4·5
5
5·5
6
6·5
2
3
4
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6
7
8
9
10

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Fig. 1

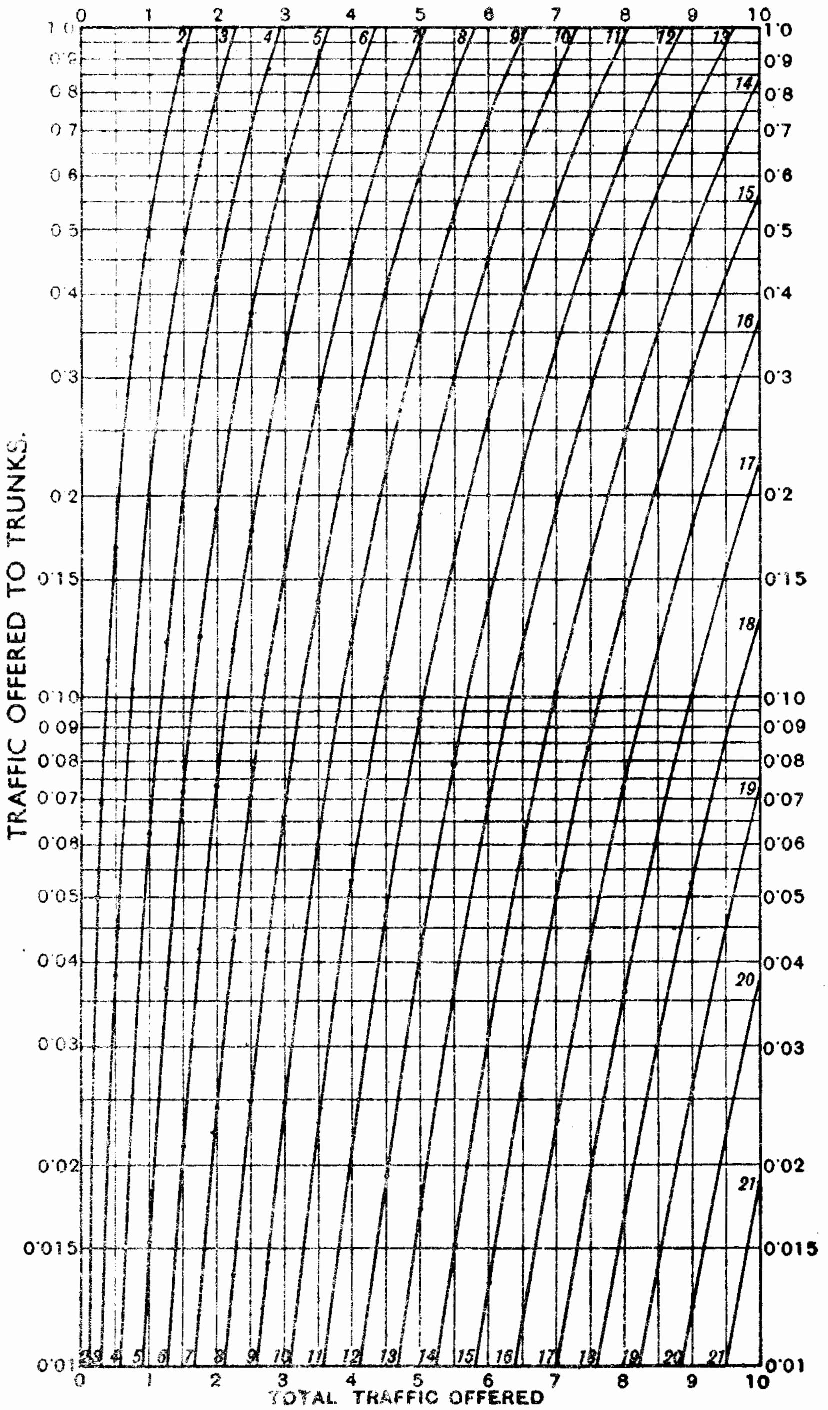

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Fig. 2

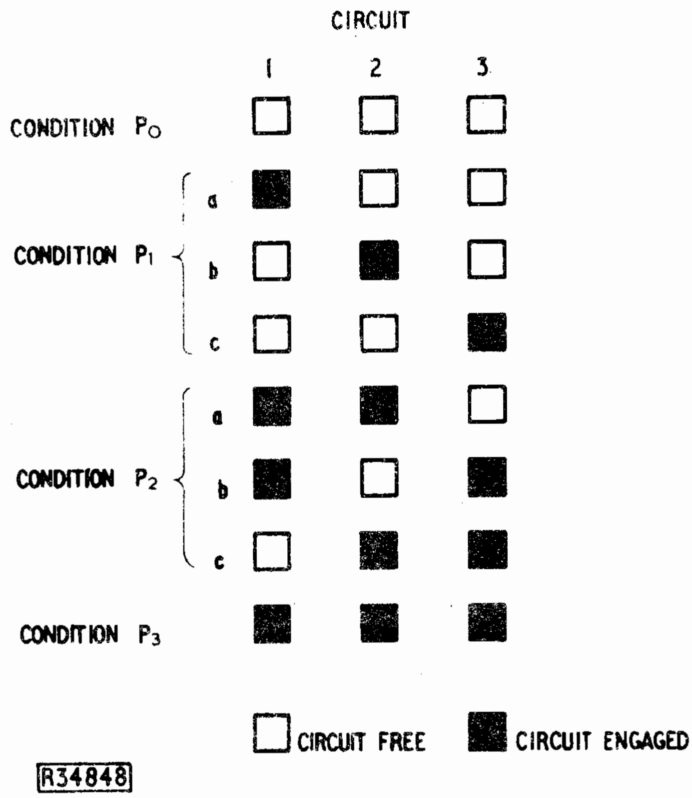
CIRCUIT
1
2
3
CONDITION P₀
CONDITION P₁
a
b
c
CONDITION P₂
a
b
c
CONDITION P₃
CIRCUIT FREE
CIRCUIT ENGAGED
R34848

Fig. 3

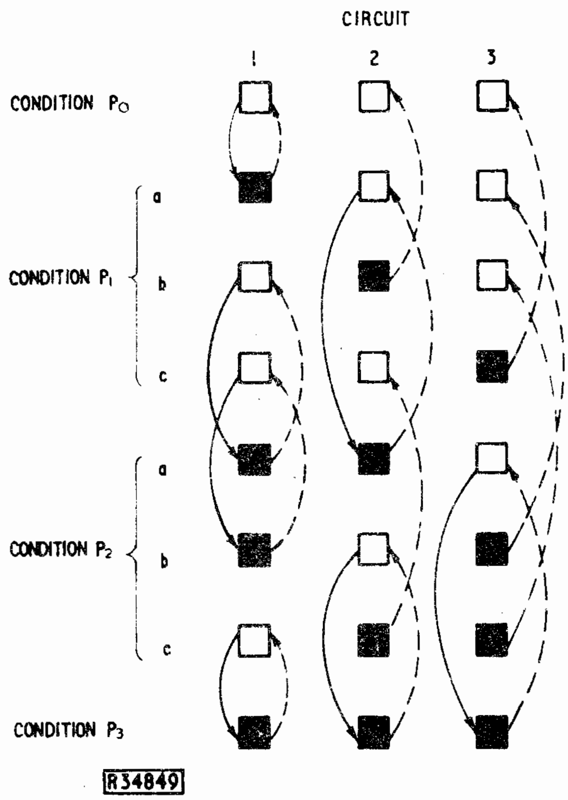
CIRCUIT
1
2
3
CONDITION P₀
CONDITION P₁
a
b
c
CONDITION P₂
a
b
c
CONDITION P₃
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Fig. 4